

---

# Universal Equivalent Circuits for All Antennas

**Steve Stearns, K6OIK**

**Consulting Engineer**

**Technical Fellow, ret.  
Northrop Grumman Corp.  
Electromagnetic Systems Laboratory  
San Jose, CA**

**[stearns@ieee.org](mailto:stearns@ieee.org)  
[k6oik@arri.net](mailto:k6oik@arri.net)**

# Speaker's Biography



- **Stephen D. Stearns**
- **Senior VP of Research, VStar Systems Inc.**
- **40 years experience in electronic systems**
  - Northrop Grumman, TRW, GTE Sylvania, Hughes Aircraft
  - Electromagnetic and signal processing systems for communications and radar surveillance, cochannel signal separation, measurement, identification, characterization, polarimetric array signal processing of ionospheric skywave signals for precision geolocating HF emitters, sensor fusion
  - Recent work: Antenna and scattering theory; Non-Foster circuits for antennas and metamaterials; antennas to radiate OAM Bessel-Vortex beams; double-reflectionless harmonic reject filters
- **FCC licenses**
  - Amateur Radio Extra Class
  - 1<sup>st</sup>-Class Radiotelephone
  - General Radio Operator License (GROL)
  - Ship Radar Endorsement
- **Education**
  - PhD Stanford – under Prof. T.M. Cover
  - MSEE USC – under Profs. H.H. Kuehl and C.L. Weber
  - BSEE CSUF – under Profs. J.E. Kemmerly and G.I. Cohn
- **10 patents**
- **More than 100 publications and presentations, both professional (IEEE) and hobbyist (Amateur Radio)**

# Topics

---

- The Smith chart
- Results from classical network theory
- Antenna impedance functions
- Low-order narrowband equivalent circuits
- Defective equivalent circuits
- Universal equivalent circuits
- Resources
  - Software
  - Antenna books

---

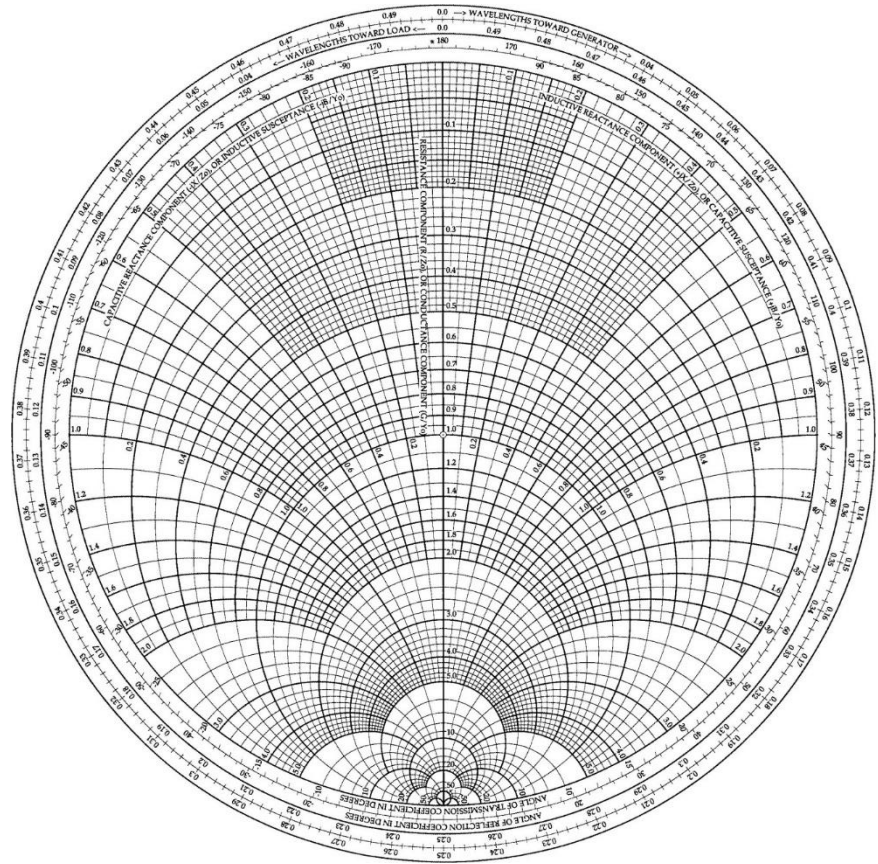
# The Smith Chart



# The Smith Chart



**Phillip Hagar Smith, 1905-1987**



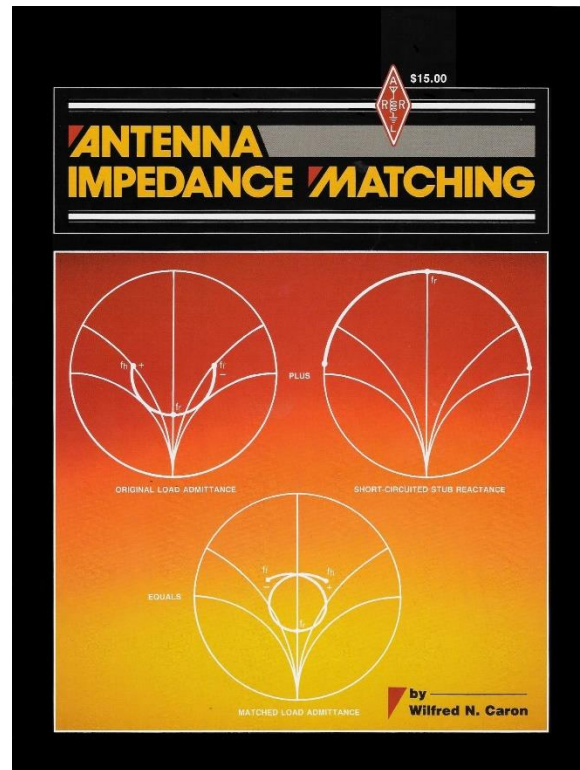
**Developed by Phillip H. Smith at Bell Labs 1936**  
**Published in *Electronics*, Jan. 1939 and Jan. 1944**  
**Mrs. Smith sold copyright to IEEE MTT-S in 2015**

# Where are Smith Charts Found?

---

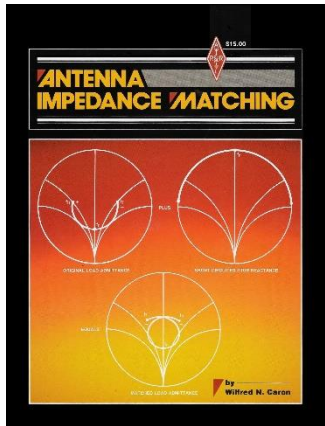
**The answer may be surprising!**

# Where are Smith Charts Found?



Book Covers

# Where are Smith Charts Found?

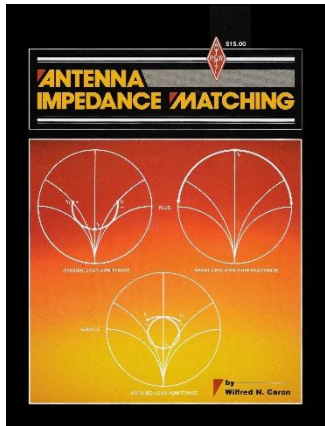


**Book Covers**

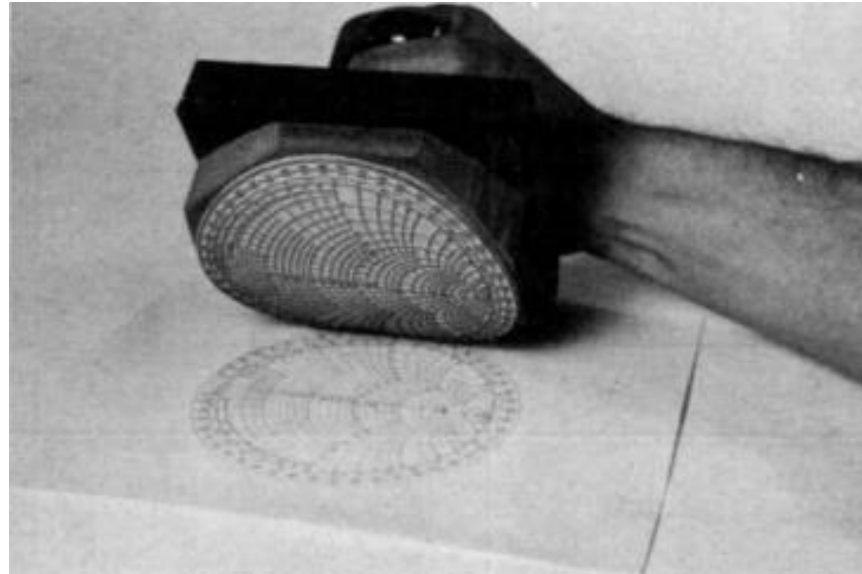


**Drink Coasters**

# Where are Smith Charts Found?



**Book Covers**



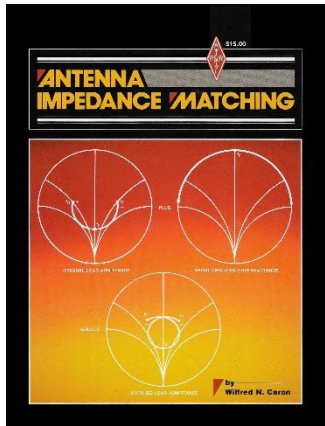
**Giant Rubber Stamps**



**Drink Coasters**



# Where are Smith Charts Found?



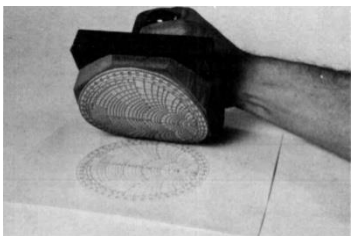
Book Covers



British Coins

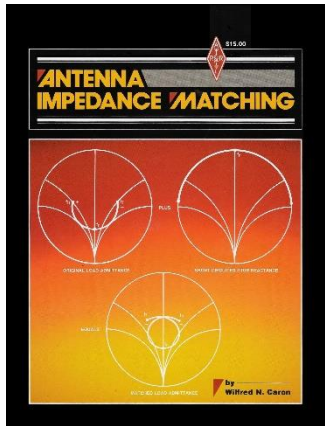


Drink Coasters



Giant Rubber Stamps

# Where are Smith Charts Found?



**Book Covers**



**British Coins**

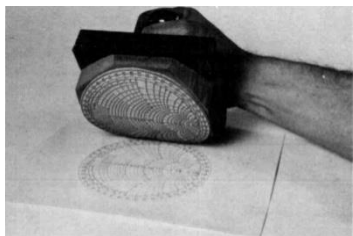


International Microwave Symposium  
17-22 May 2015, Phoenix

**IEEE Conference Logo**

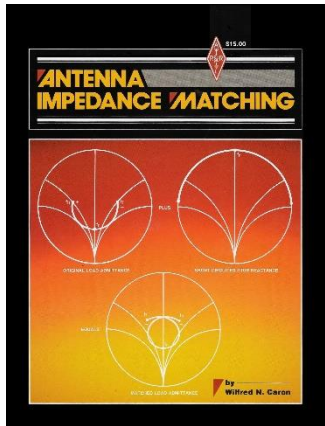


**Drink Coasters**



**Giant Rubber Stamps**

# Where are Smith Charts Found?

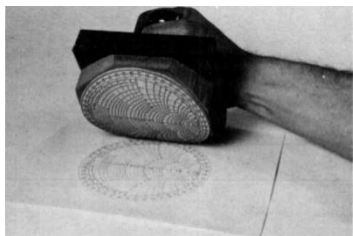


**Book Covers**



International Microwave Symposium  
17-22 May 2015, Phoenix

**IEEE Conference Logo**



**Giant Rubber Stamps**



**Tatoos in Berkeley**



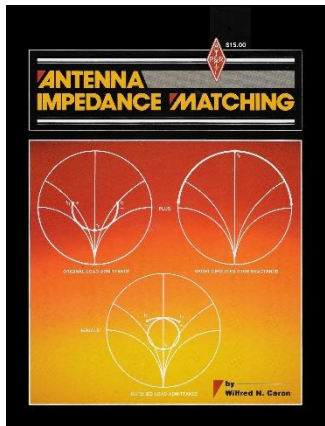
**British Coins**



**Drink Coasters**



# Where are Smith Charts Found?

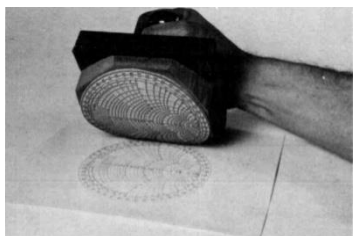


Book Covers



International Microwave Symposium  
17-22 May 2015, Phoenix

IEEE Conference Logo



Giant Rubber Stamps



Bicycling in Los Angeles!



British Coins

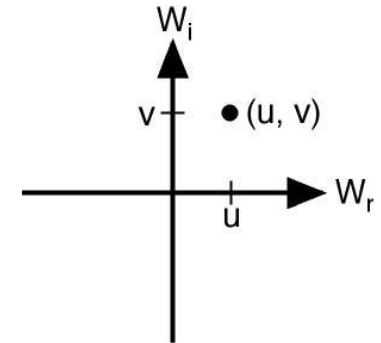
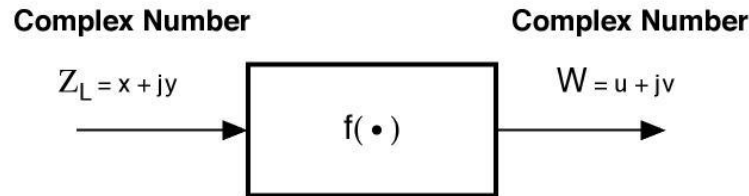
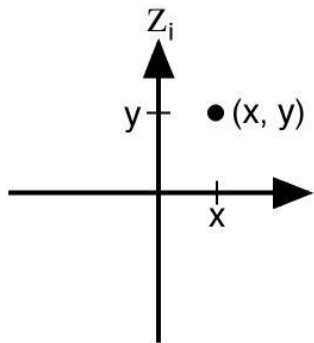


Drink Coasters



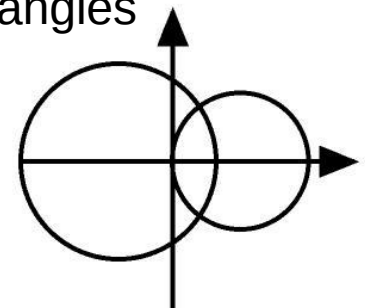
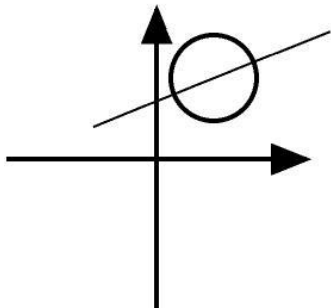
Tatoos in Berkeley

# Complex Functions

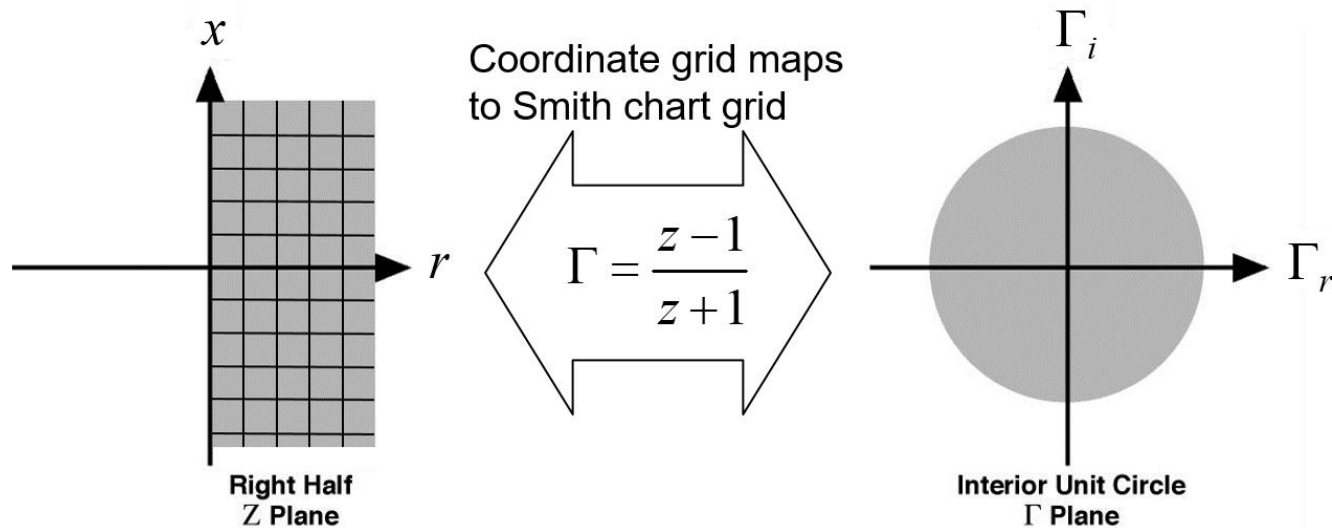


## ■ Basic types of complex functions

- Global Properties
  - Linear – lines map to lines
  - Bilinear – circles map to circles
- Local Properties
  - Conformal – right angles map to right angles

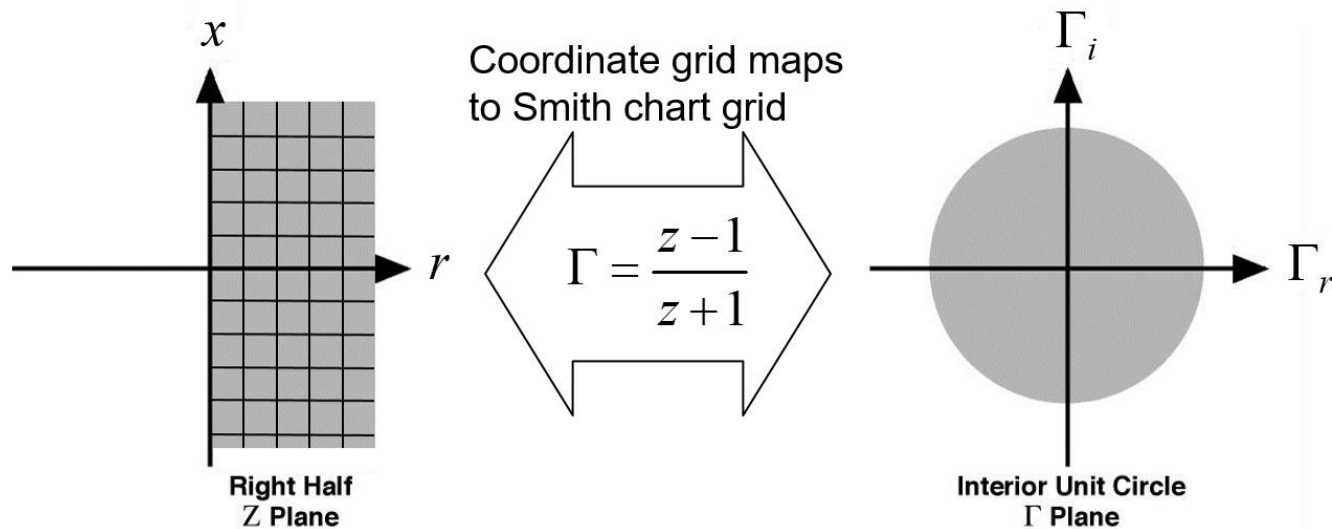


# Mathematical Basis of the Smith Chart



- A bilinear, conformal mapping of complex number to complex number
- Maps normalized impedance  $z$  to complex reflection coefficient  $\Gamma$
- Maps right half of impedance and admittance planes to interior of unit circle in  $\Gamma$  plane

# Smith Chart Coordinate Grid



$$\Gamma_r + j\Gamma_i = \frac{(r-1) + jx}{(r+1) + jx}$$

$$\Gamma_r = \frac{r^2 + x^2 - 1}{(r+1)^2 + x^2}$$

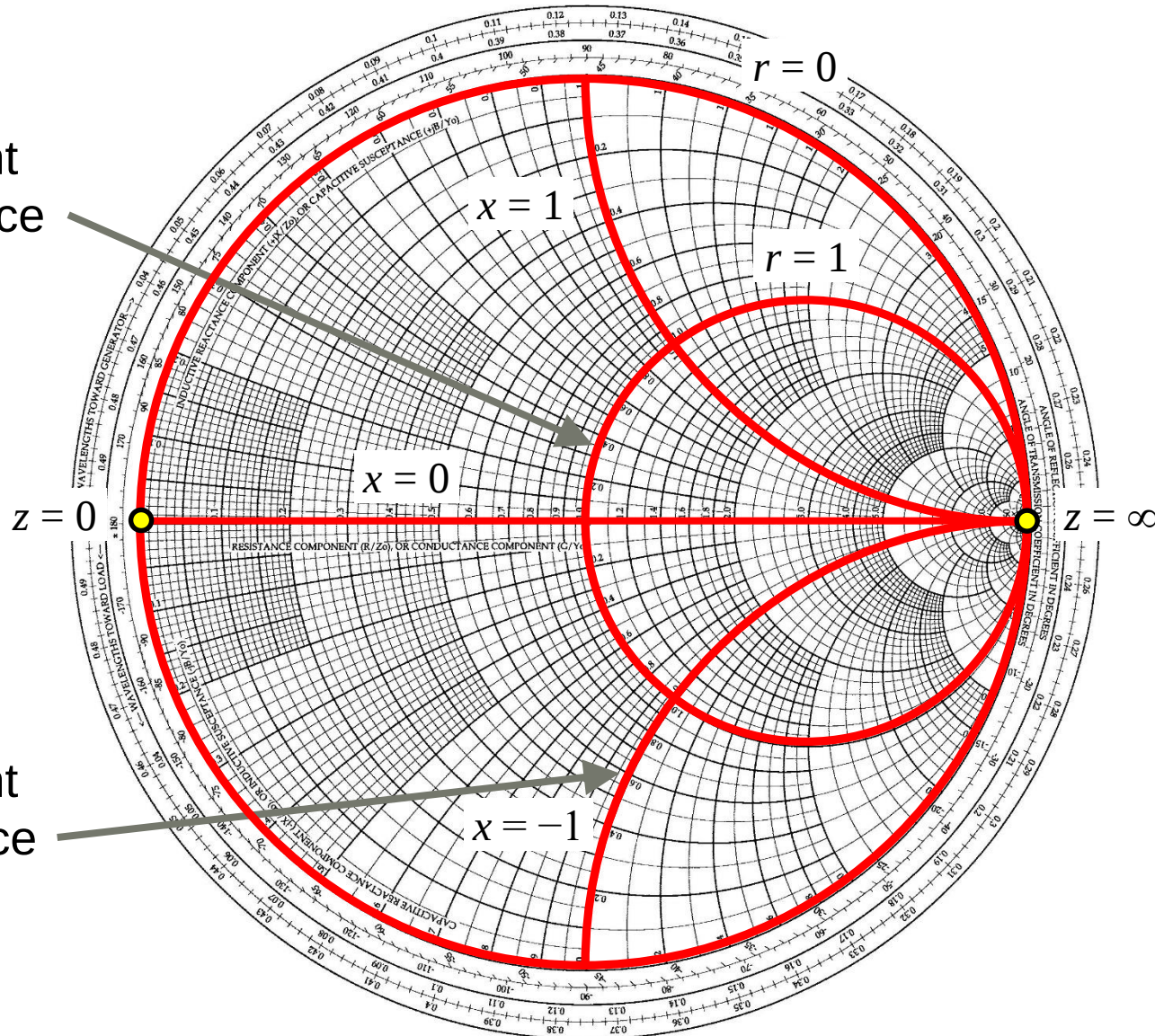
$$\Gamma_i = \frac{2x}{(r+1)^2 + x^2}$$



# Impedance Coordinates

Constant  
resistance  
circles

Constant  
reactance  
arcs

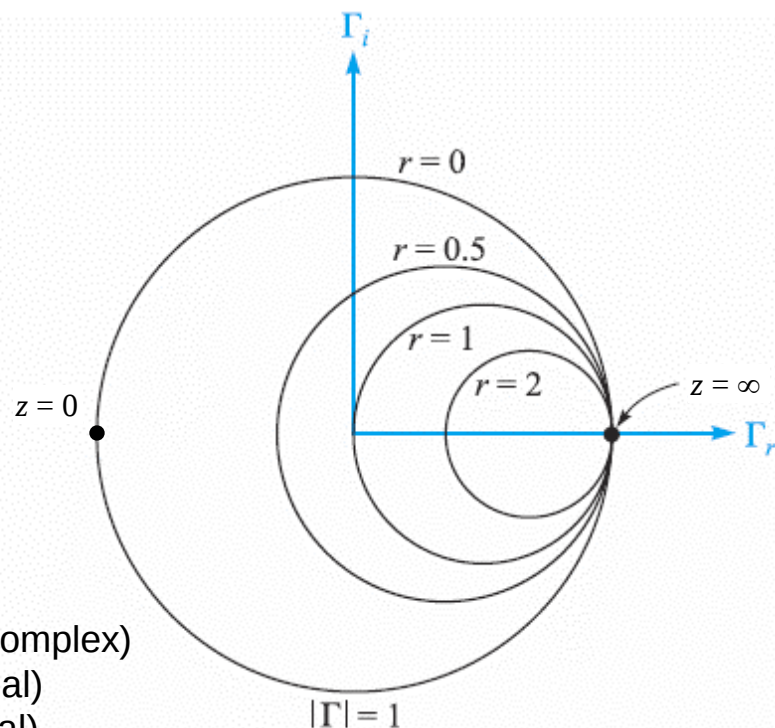
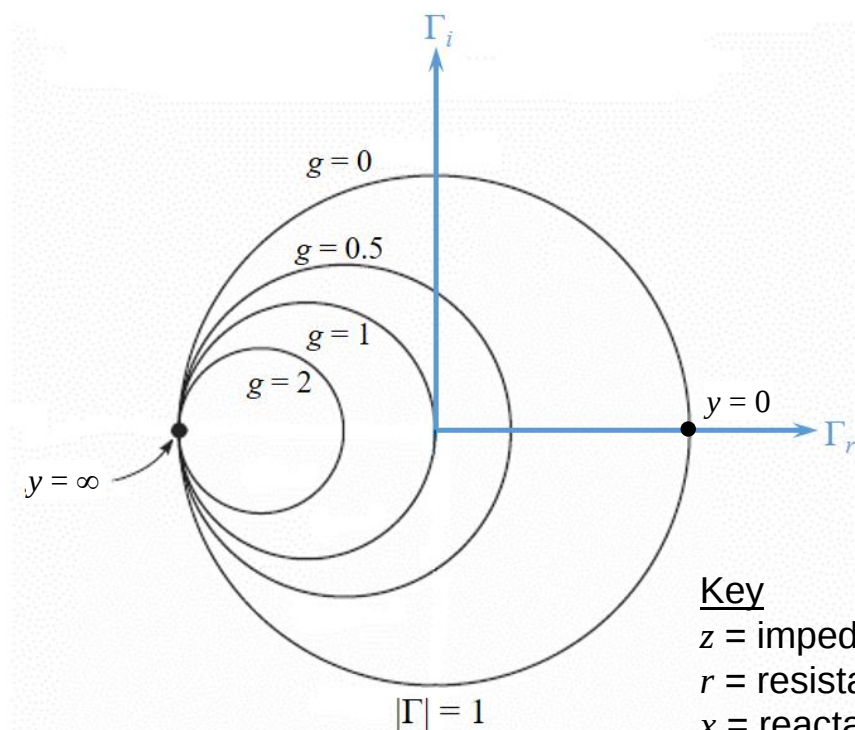






# Conductance and Resistance Circles

## Admittance and Impedance Zero and Infinity Points

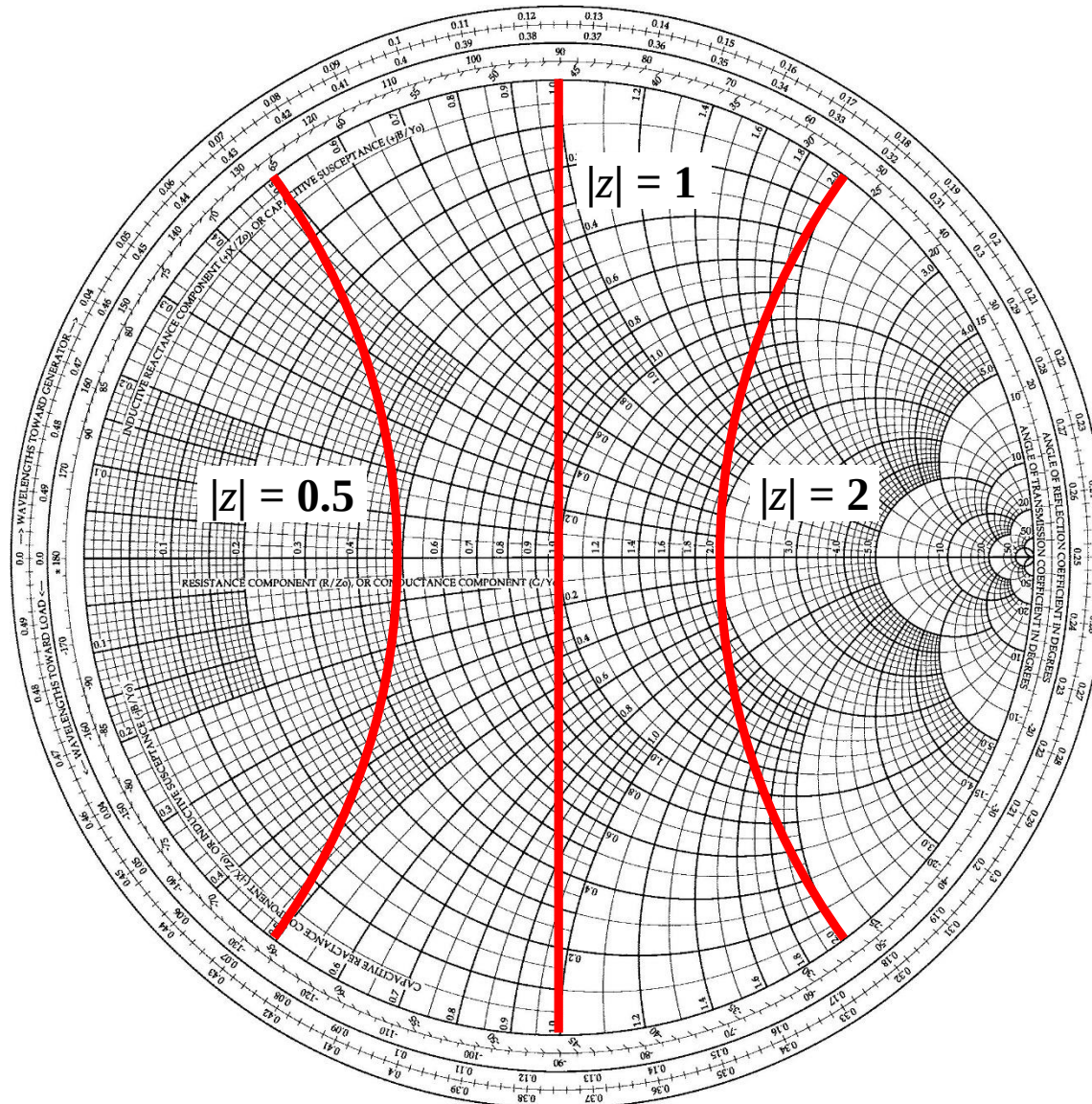


### Key

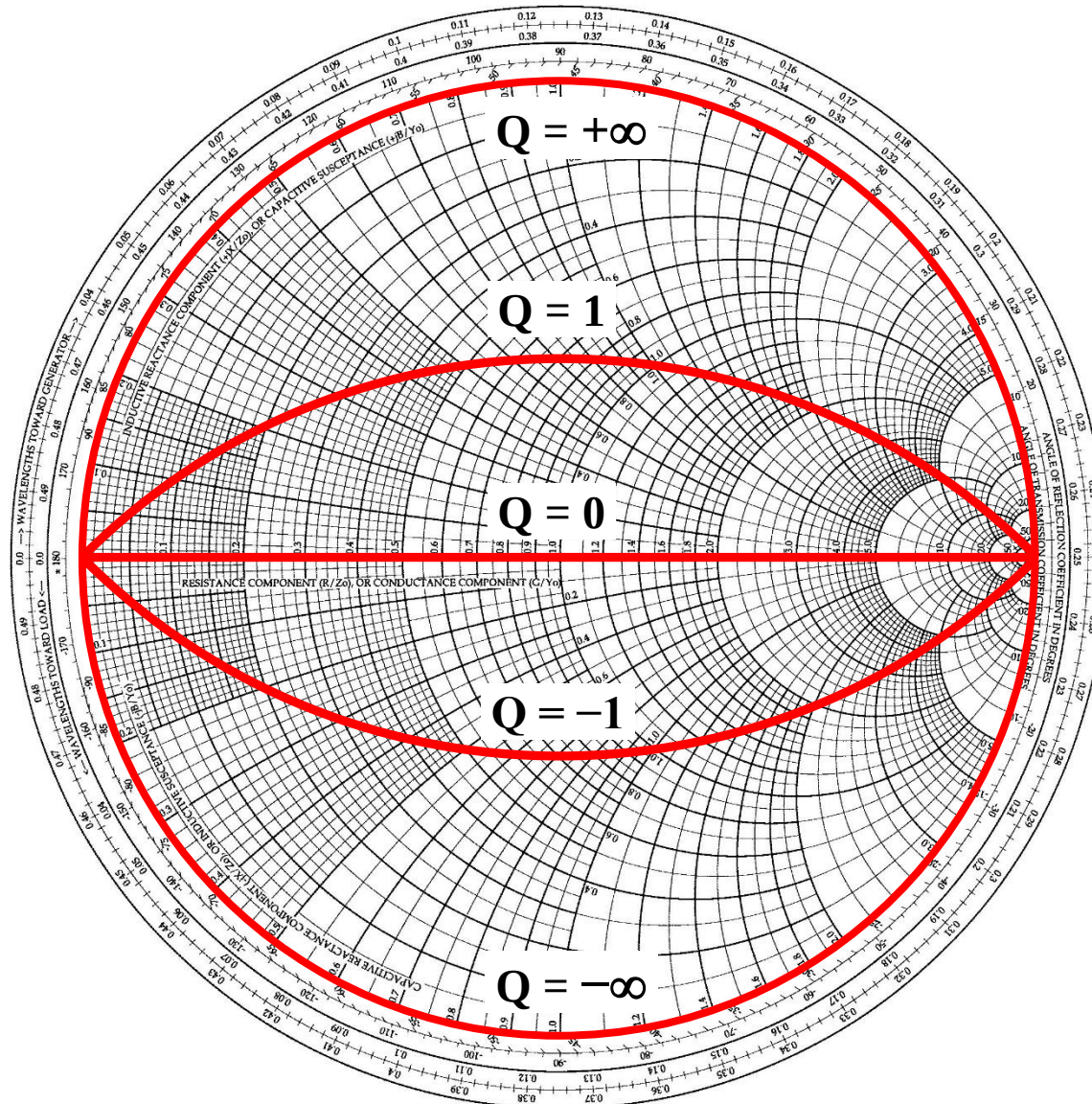
$z$  = impedance (complex)  
 $r$  = resistance (real)  
 $x$  = reactance (real)  
 $y$  = admittance (complex)  
 $g$  = conductance (real)  
 $b$  = susceptance (real)  
 $\Gamma$  = reflection coefficient (complex)  
 $\Gamma_r$  = real part of  $\Gamma$  (real)  
 $\Gamma_i$  = imaginary part of  $\Gamma$  (real)  
*lower case* = normalized



# Constant Immittance Magnitude Arcs

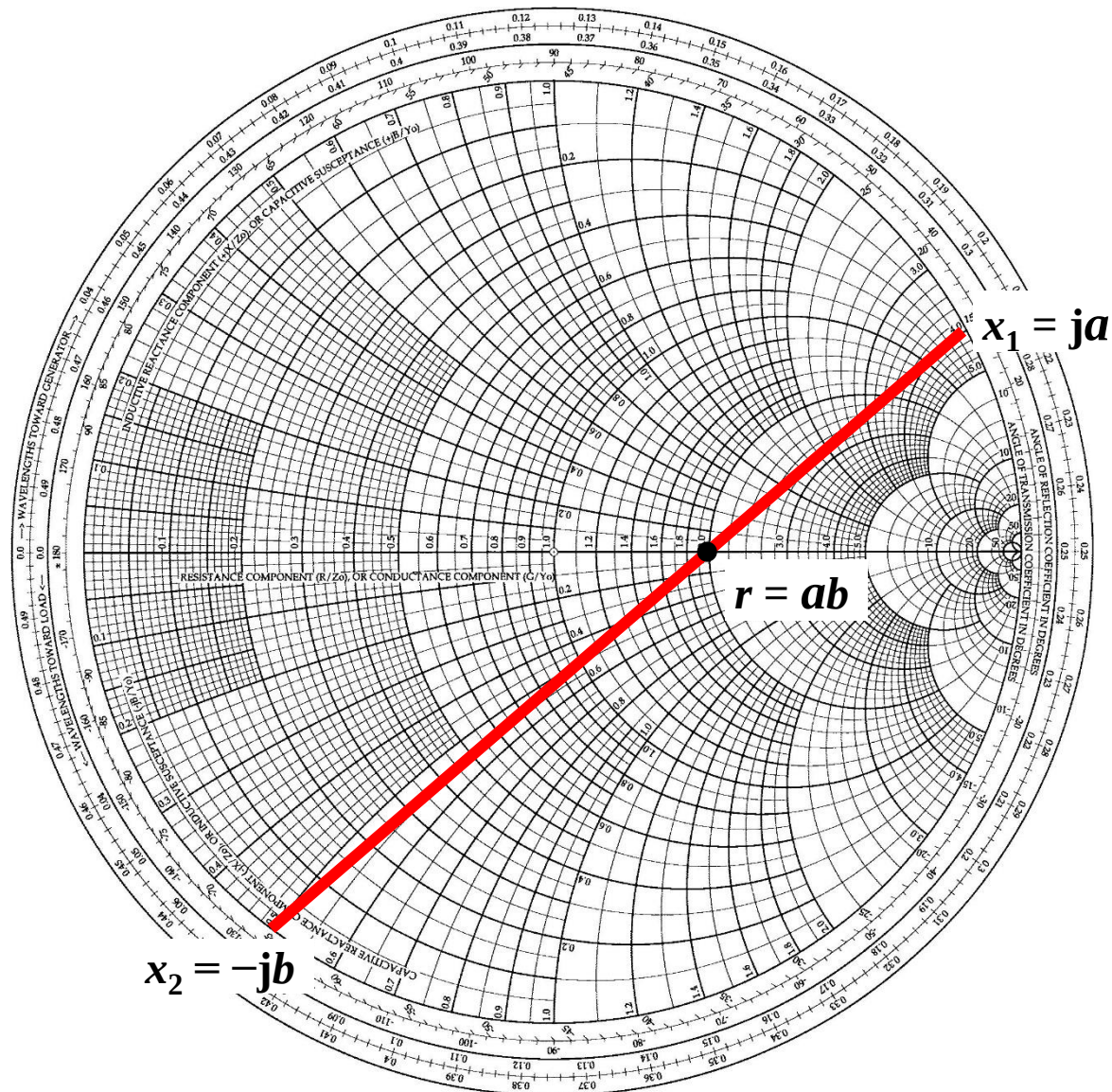


# Constant Q (Immittance Phase) Arcs

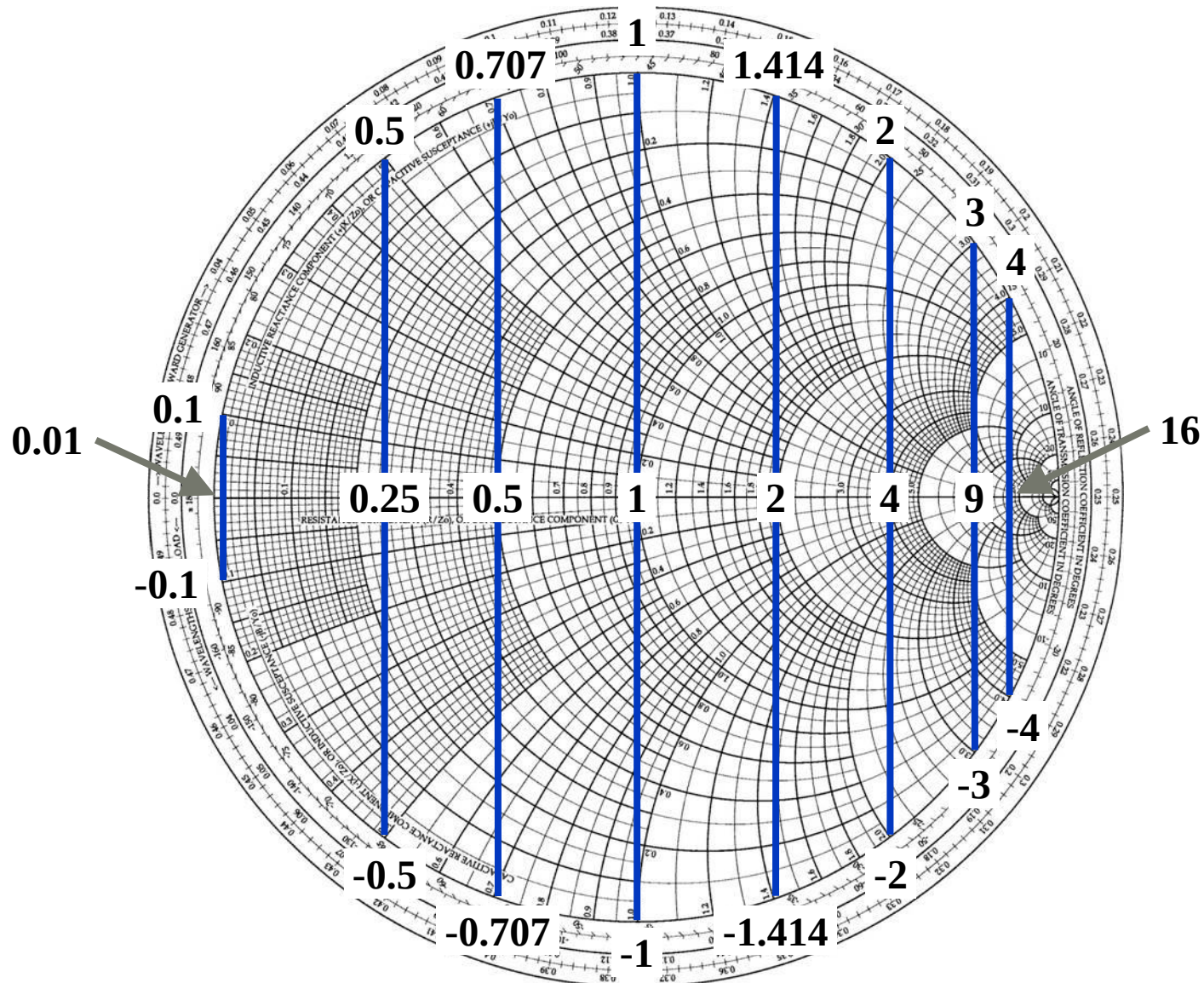




# Multiplication and Division

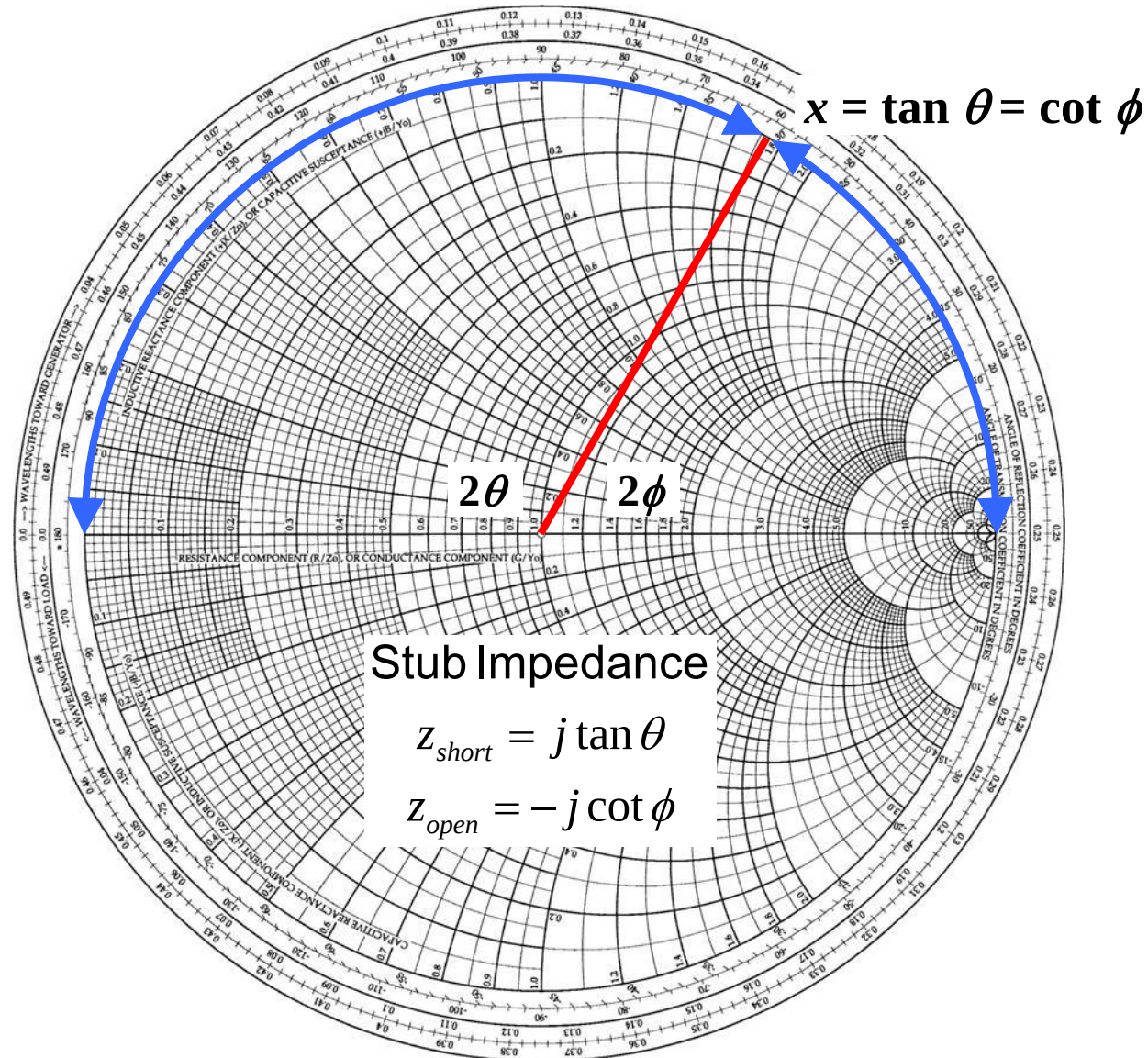


# Squares and Square Roots



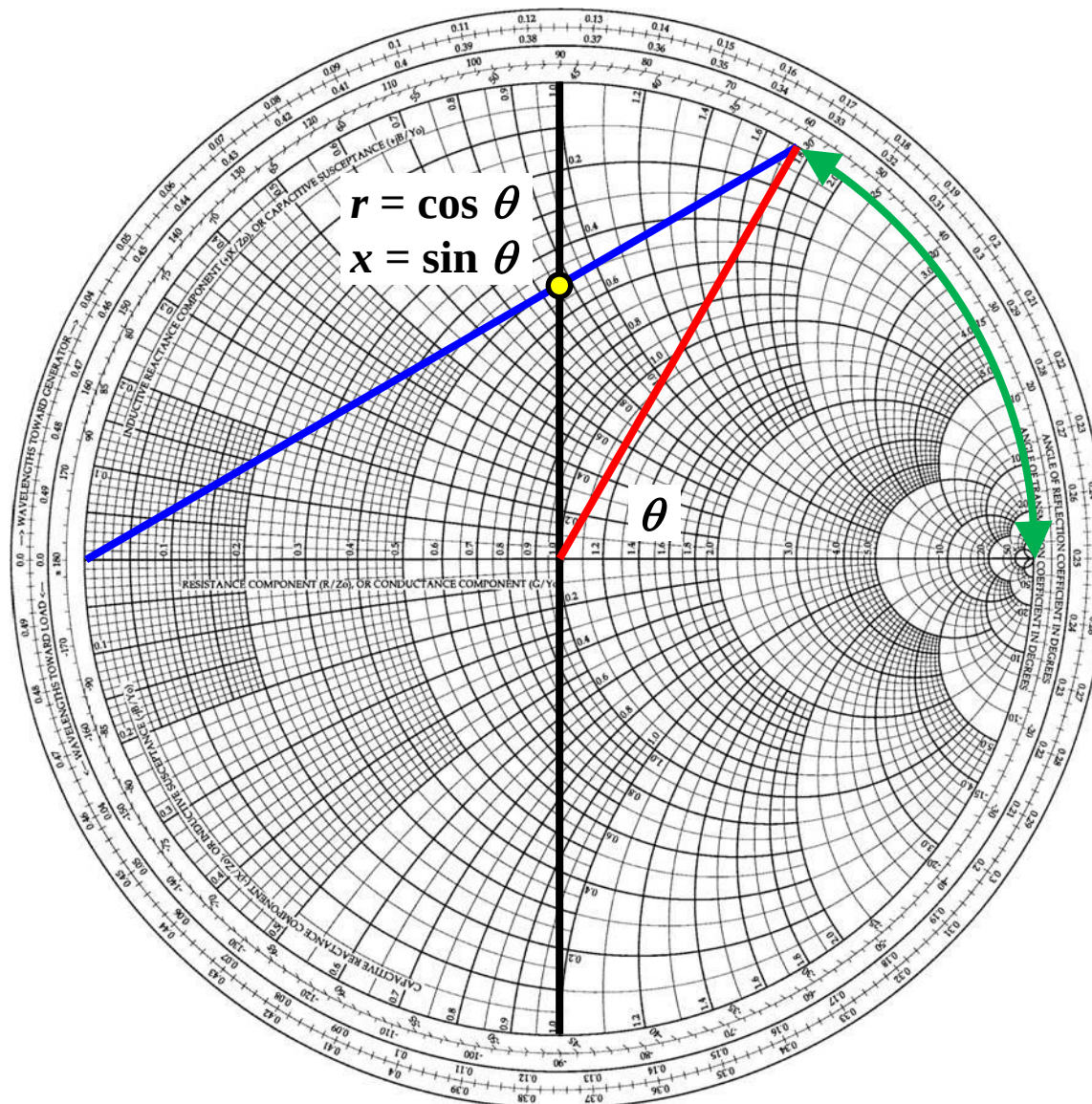


# Tangents and Cotangents





# Sines and Cosines



---

# **Classical Network Theory**

# Key Dates in Classical Network Theory

---

- 1893** “Impedance” – A.E. Kennelly
- 1893** AC circuit theory using complex numbers or phasors, “reactance” – C.P. Steinmetz
- 1923-1924** Reactance Theorems – O.J. Zobel, R.M. Foster
- 1924** Lossless impedance synthesis by partial fractions – R.M. Foster
- 1926** Lossless impedance synthesis by continued fractions – W. Cauer
- 1931** Passive impedance synthesis using RLCM – O. Brune
- 1937** Passive impedance synthesis using a single resistor – S. Darlington
- 1946** Passive n-port synthesis using RLCM – Y. Oono
- 1949** Passive synthesis using RLC without transformers – R. Bott & R.J. Duffin
- 1957** *Synthesis of Passive Networks* – E.A. Guillemin
- 1958** *Network Synthesis* – D.F. Tuttle
- 1962** *Linear Active Network Theory* – L. de Pian
- 1973** *Network Analysis and Synthesis* – B.D.O. Anderson and S. Vongpanitlerd
- 2009** All singular elements found – A.M. Soliman
- 2002-2010** Realizability, synthesis, and stability of non-Foster active networks – S.E. Sussman-Fort, S.D. Stearns, others

# Pioneers of Electric Network Theory



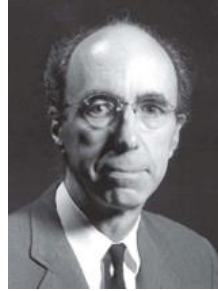
Ronald Martin Foster  
1896-1998



Wilhelm Cauer  
1900-1945



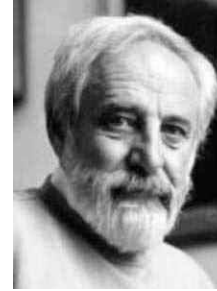
Otto Walter Heinrich  
Oscar Brune  
1901-1982



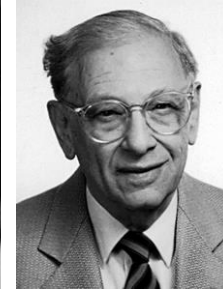
Sidney Darlington  
1906-1997



Richard James Duffin  
1909-1996



Raoul Bott  
1923-2005



Herbert Jacob Carlin  
1917-2009



Dante Ciriaco Youla  
1925-2021



Arthur Edwin Kennelly  
1861-1939



Charles Proteus Steinmetz  
1865-1923



George Ashley Campbell  
1870-1954



Otto Julius Zobel  
1887-1970



Ernst Adolf Guillemin  
1898-1970



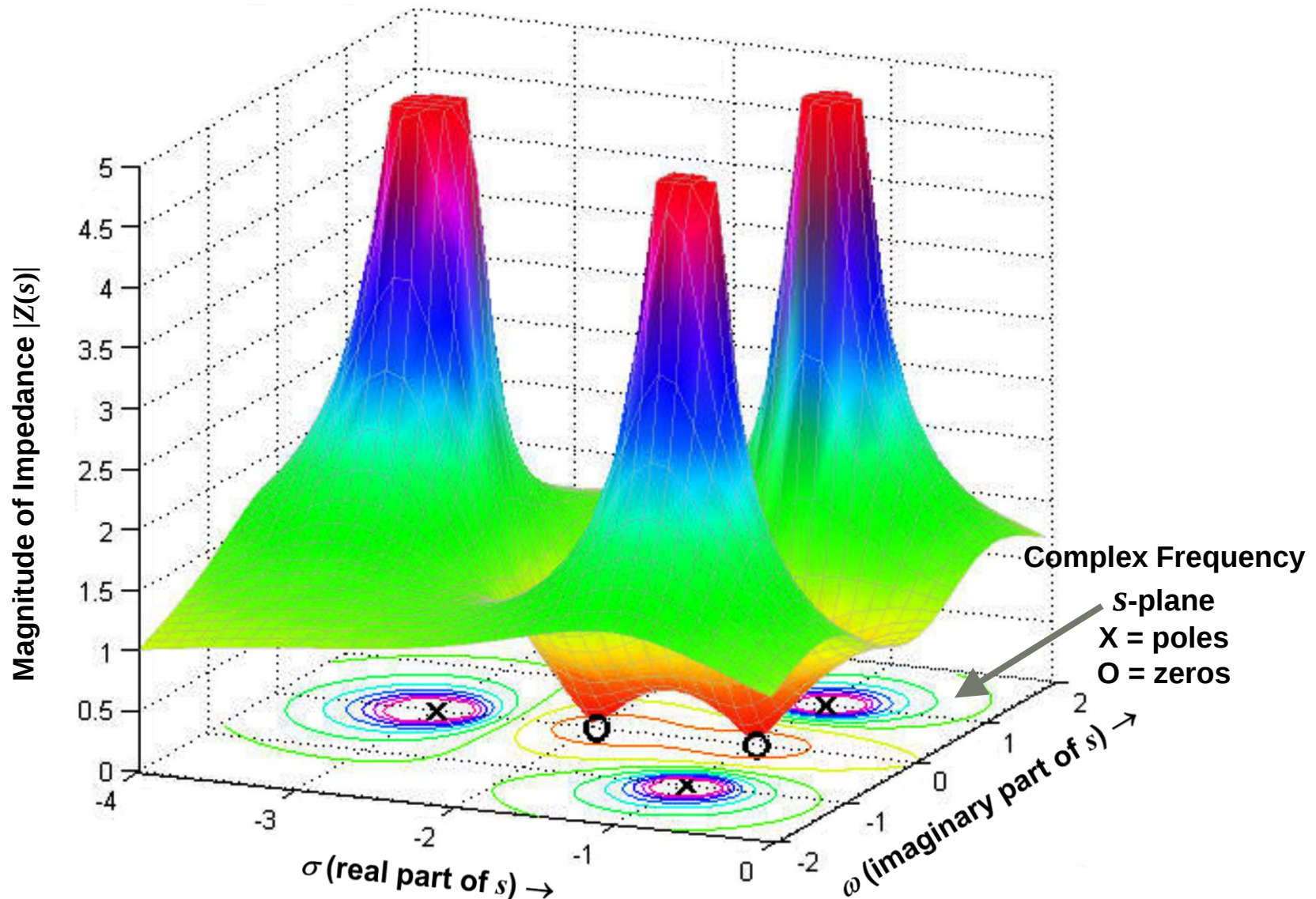
Mac Elwyn Van Valkenburg  
1921-1997



Norman Balabanian  
1922-2009



# Poles, Zeros, and Complex Frequency



# Terminology: Impedance, Admittance, Immittance

- Generalizes “resistance” for *a-c* circuits – an impedance is a complex number – a point in a complex number plane
- Expressed in Cartesian or polar form

$$Z = R + jX = |Z| \angle \theta$$

Real part      Imaginary part      Magnitude      Phase angle  
Resistance      Reactance

- Admittance is the inverse of impedance

$$\frac{1}{Z} = Y = G + jB = |Y| \angle \theta$$

Real part      Imaginary part  
Conductance      Susceptance

- Ohm's Law

$$\left( \frac{E}{I | Z} \right) \quad \left( \frac{I}{E | Y} \right)$$

# Immittance (Impedance & Admittance) Functions

- Immittance functions of passive devices, elements, and networks are “positive-real” functions of complex frequency  $s = \sigma + j\omega$
- Analytic in the right half plane – no poles or zeros there
- Poles and zeros can exist only on  $j\omega$  axis and in the left half plane
- Immittance functions of lumped RLC networks are rational functions

$$Z(s) = \frac{as^3 + bs^2 + cs + d}{es^2 + fs}$$

- All coefficients positive, no middle terms missing
- Degrees of numerator and denominator polynomials differ by at most 1
- If the degrees are the same, the network has losses
- The magnitude of an immittance function of a passive element or network cannot increase faster than  $f$  (1<sup>st</sup> power of frequency) nor decrease faster than  $1/f$  (inverse frequency)
- For lossless elements, devices, and networks, the slopes of reactance and susceptance vs frequency are strictly positive at all frequencies (except discontinuities)

# An Important Fact about Impedance Functions

---

**The real and imaginary parts of a passive immittance (impedance or admittance) cannot be specified independently. One determines the other.**

- Real and imaginary parts are related by Poisson/Schwarz integrals
- A resistance is generally accompanied by reactance or is not passive
- Realizable circuits use only passive components



# Relation Between Real and Imaginary Parts

- **Poisson/Schwarz integrals**

- AKA “Hilbert transform” or Kramers-Kronig relations

$$R(\omega) = \frac{2}{\pi} \int_0^{\infty} \frac{u X(u)}{\omega^2 - u^2} du + R_0$$

← A positive constant

$$X(\omega) = \frac{-2\omega}{\pi} \int_0^{\infty} \frac{R(u)}{\omega^2 - u^2} du + X_0(\omega)$$

← An analytic function

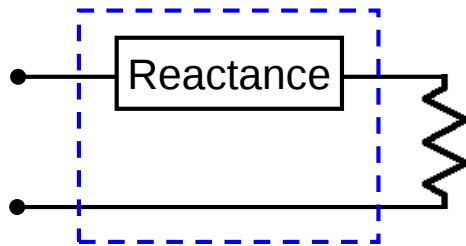
- **A consequence**

- Letting  $R(\omega) = R\omega^2$ , we find that  $X(\omega)$  does not exist
- Passive square-law resistors are an impossibility

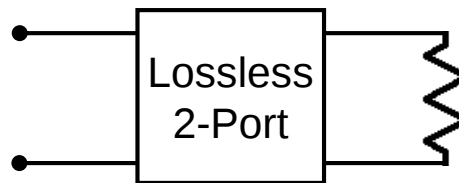
The real and imaginary parts of a passive impedance are not independent.

# Darlington Forms (1937, 1939)

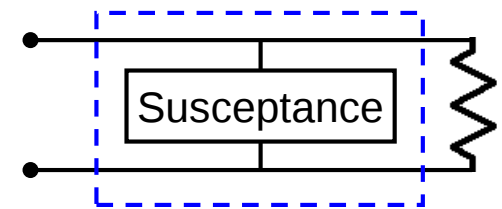
- Every positive real, rational immittance function can be realized as a reactance 2-port terminated by a resistor



Not series!



Darlington form



Not parallel!

- Antennas can be represented by equivalent circuits in Darlington form, over any bandwidth
- 2-port antenna emulators are based on Darlington forms

S. Darlington, "Synthesis of Reactance 4-Poles which Produce Prescribed Insertion Loss Characteristics," *Journal of Mathematics and Physics*, vol. 18, no. 4, pp. 257-353, April 1939.

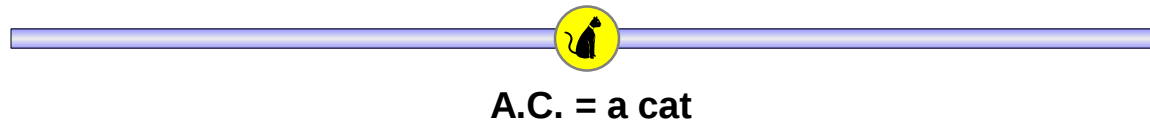
Every antenna has an equivalent circuit over arbitrarily wide bandwidth, in Darlington form.

---

# **Antenna Impedance Functions**

**On the Smith Chart**

# Dipole Antenna in Free Space



- A dipole antenna is symmetric and center fed
- A dipole's feedpoint impedance is a complex-valued function of frequency, length, and diameter

$$Z(f, L, d) = R(f, L, d) + jX(f, L, d)$$

- 1<sup>st</sup> resonance occurs when length is a little less than a half wavelength

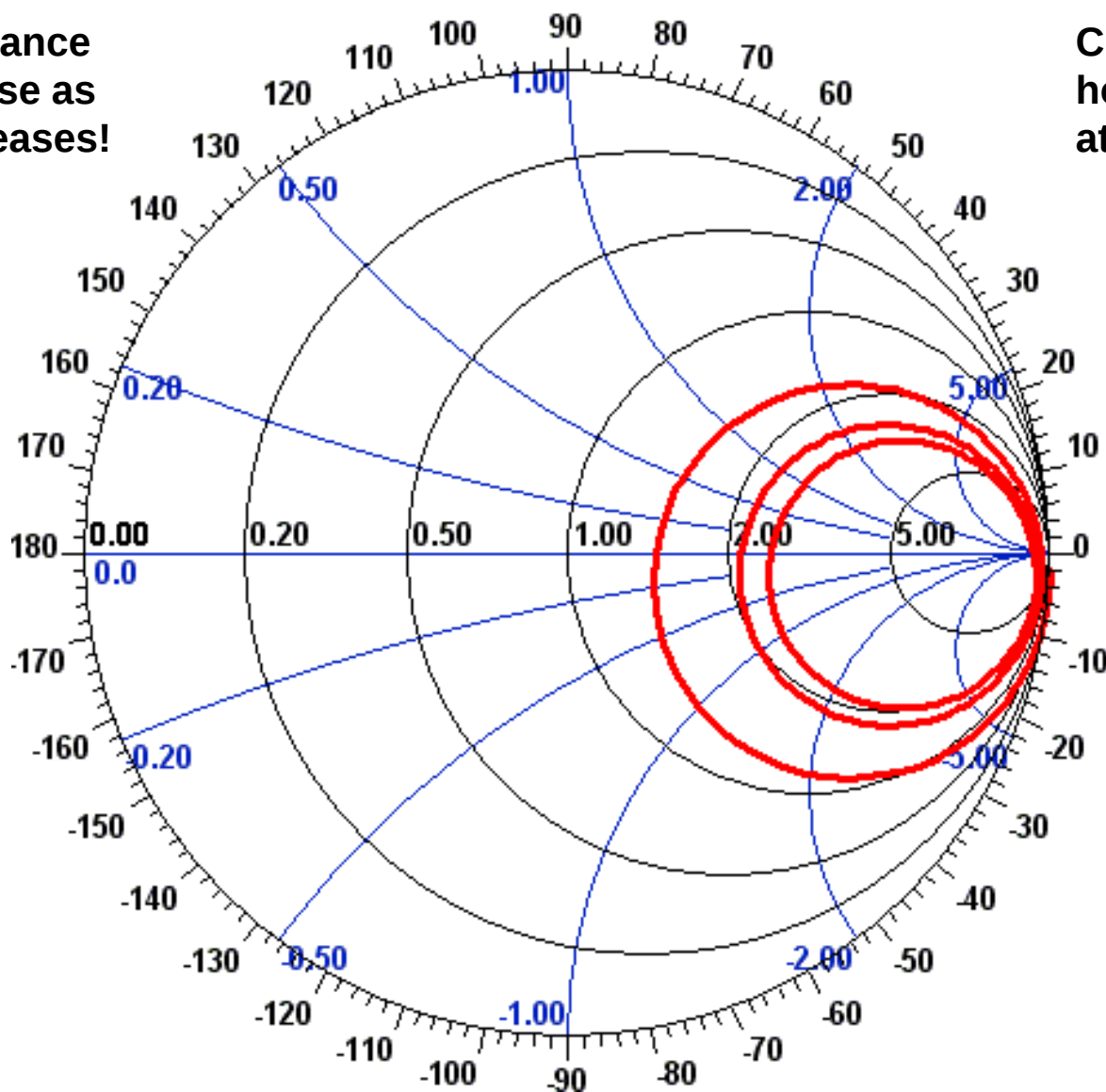
$$L = \frac{K\lambda}{2} = \frac{Kc}{2f}$$



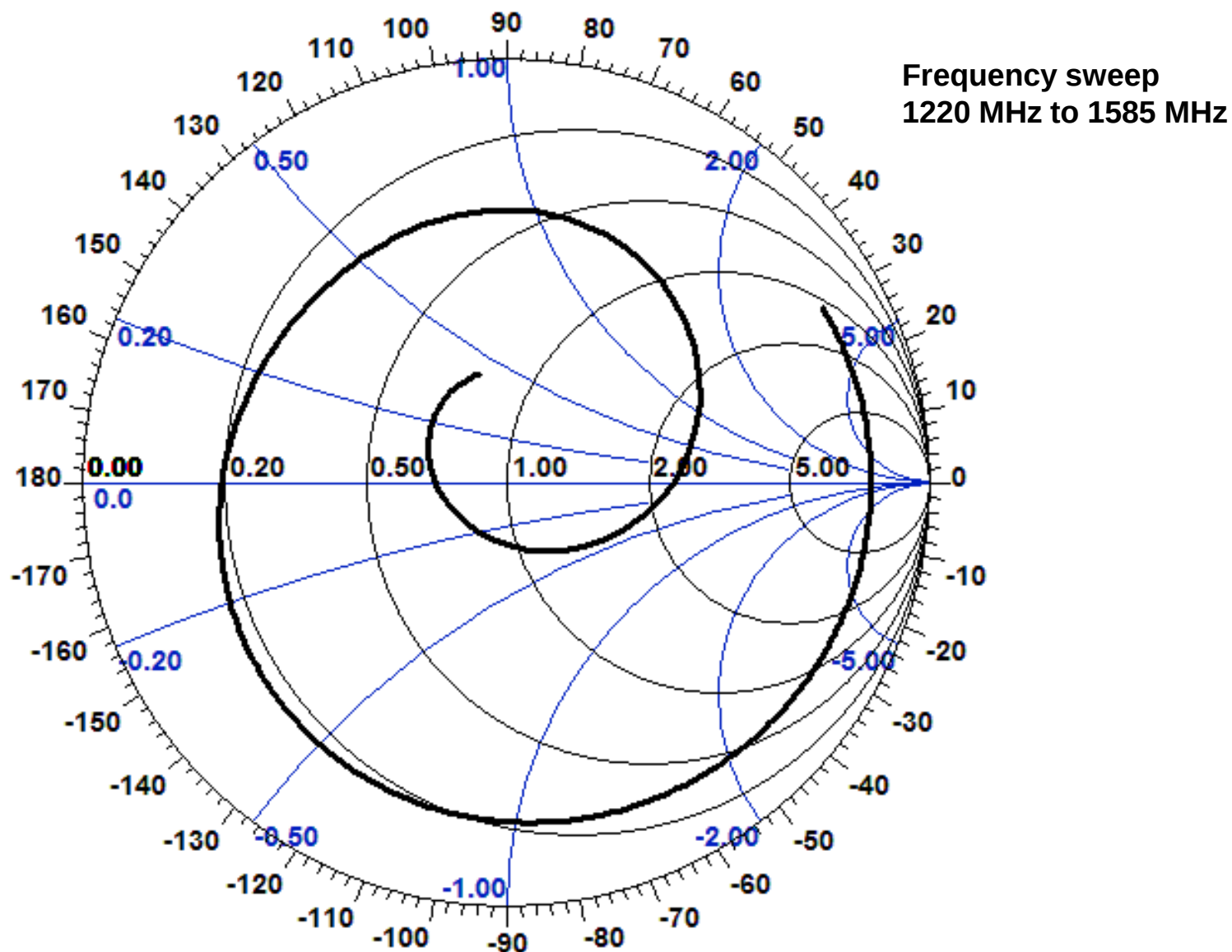
# Example 1: 98.4-ft Thin-Wire Dipole ( $L/d = 11,000$ ) from 1 MHz to 30 MHz

Antenna impedance  
curves clockwise as  
frequency increases!

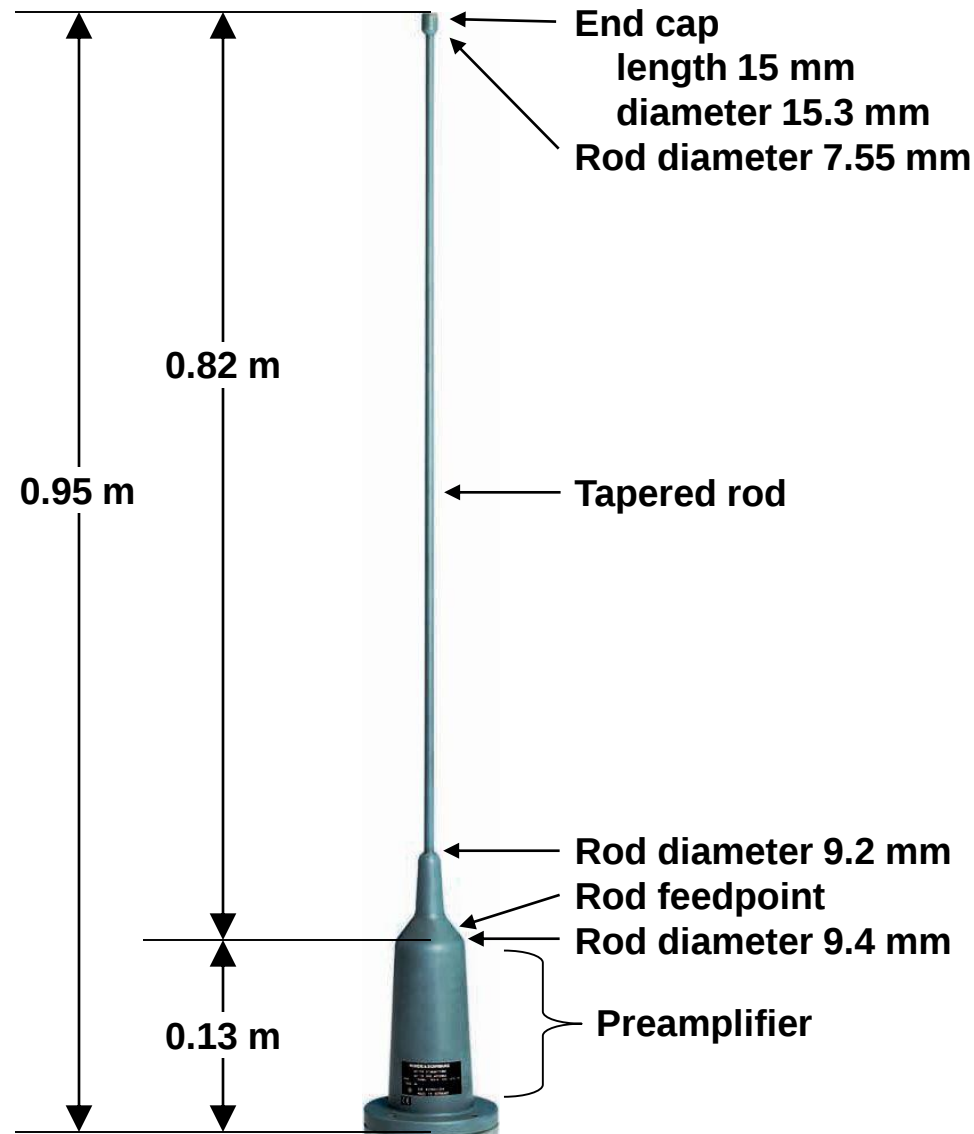
Curve starts on  
horizontal axis  
at  $f = 0$



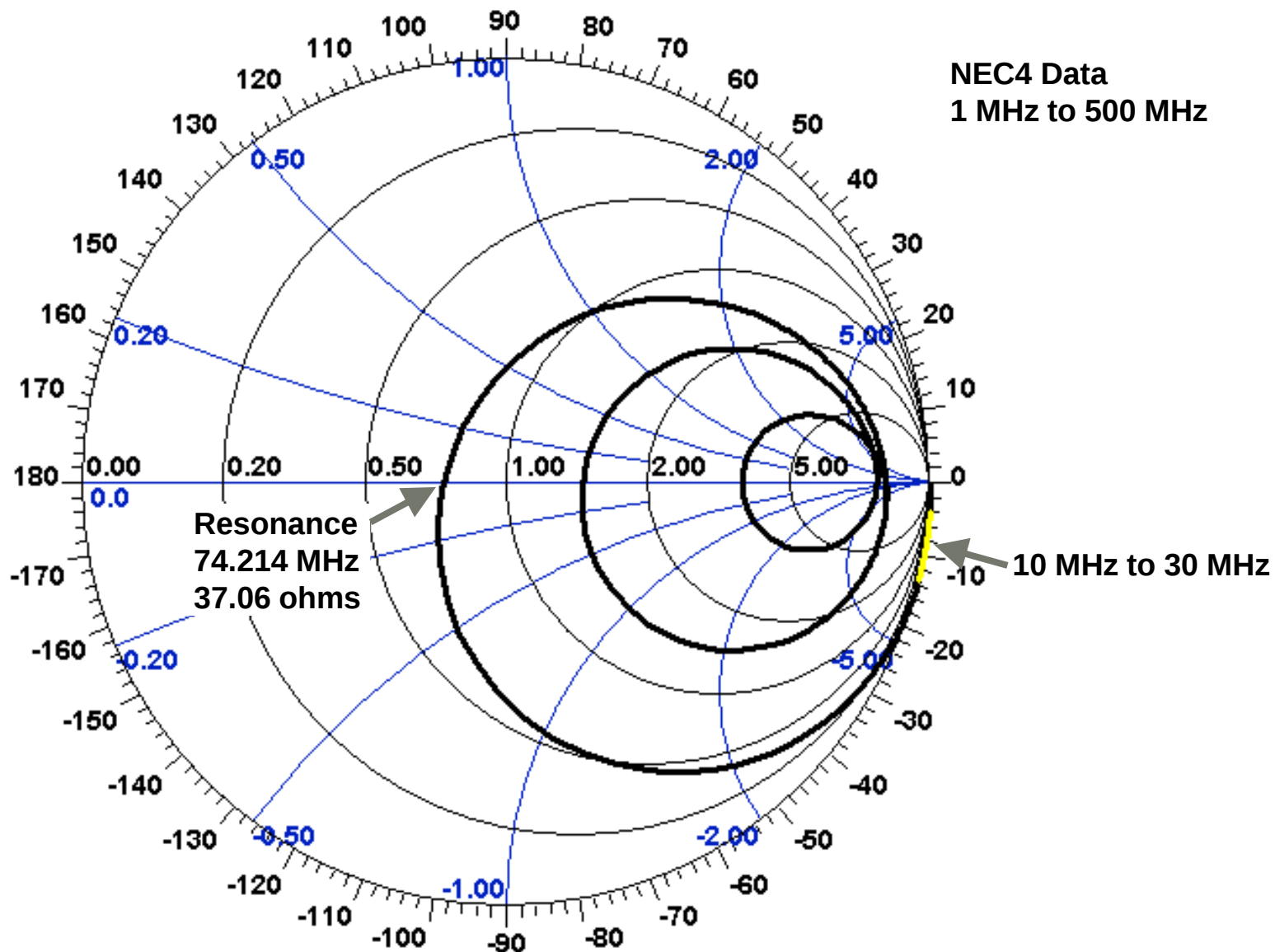
## Example 2: GPS Antenna



# Example 3: Rohde & Schwarz HE010 Active Monopole



# Example 3: R&S HE010 Monopole Impedance



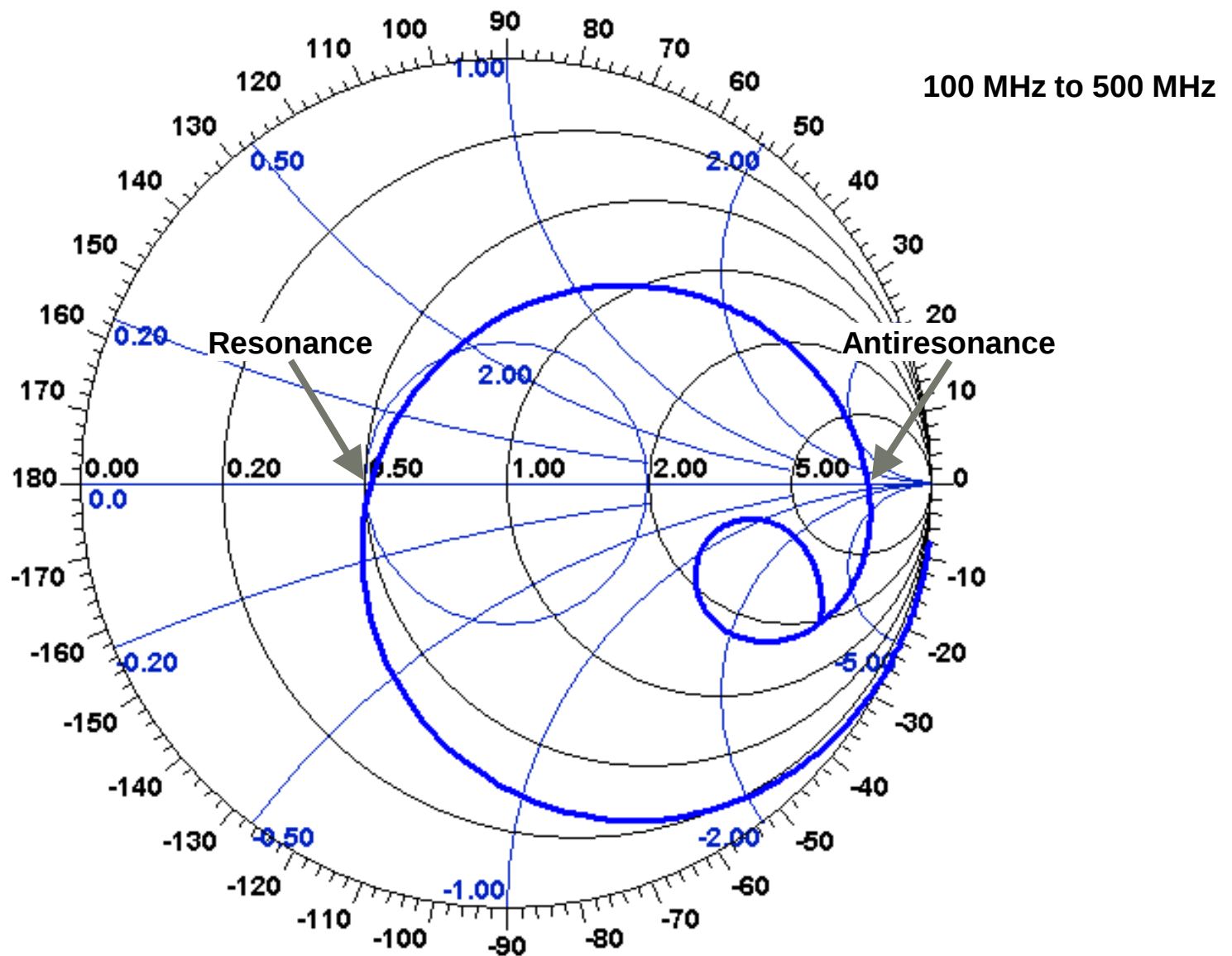
# Typical Spy Plane

## Tupolev Tu-16R Badger

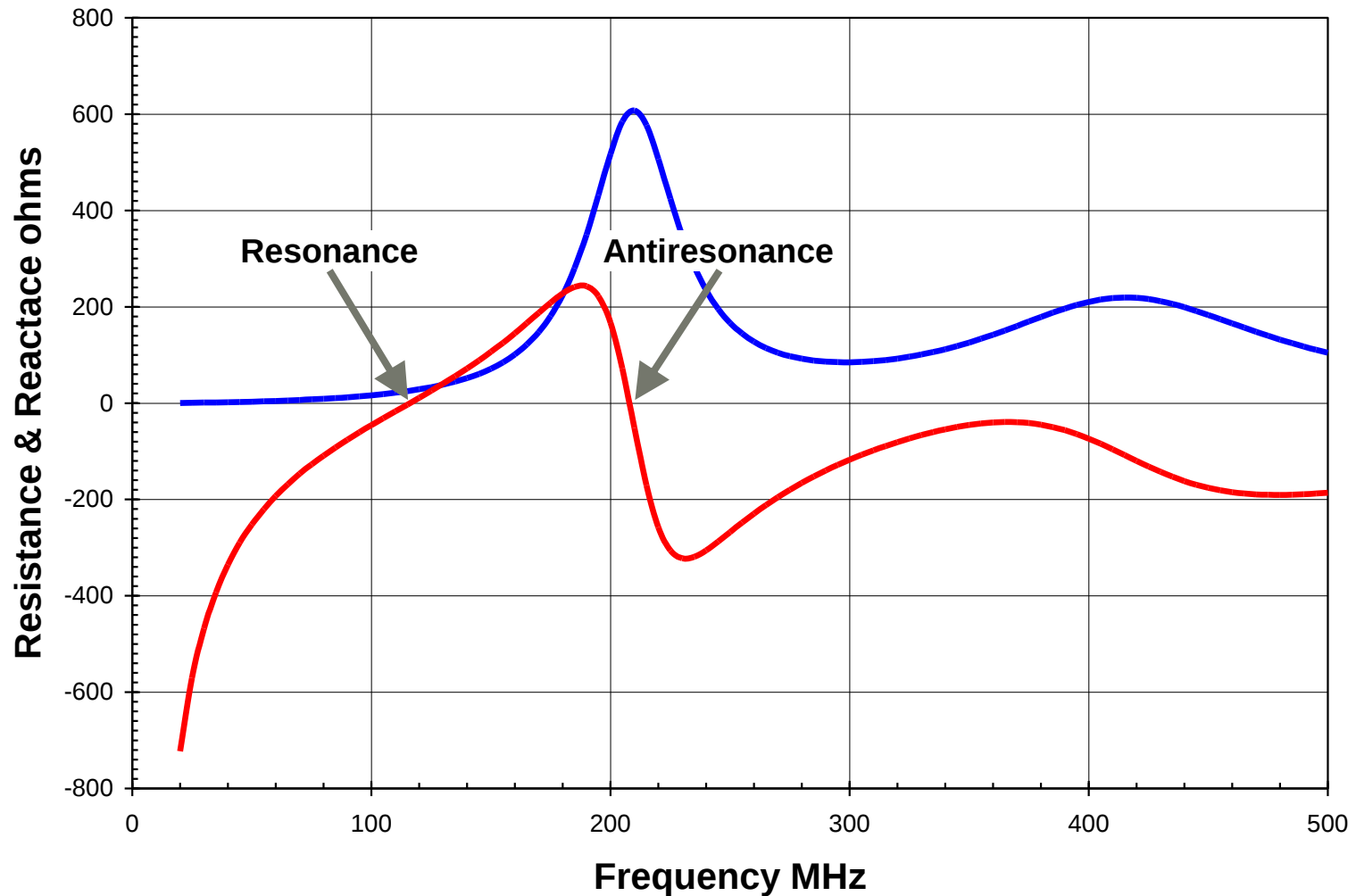




## Example 4: Bent-Blade Antenna



## Example 4: Bent-Blade Antenna Feedpoint Impedance



# Example 5: VHF-UHF Discone Antenna

## All About the Discone Antenna: Antenna of Mysterious Origin and Superb Broadband Performance

*Learn about the development, history and some applications of a discone antenna.*

Steve Stearns, K6OIK

**T**he frequency bandwidths demanded by high-definition television have considerable range....<sup>1</sup> With these prescient words, Philip S. Carter of RCA opened a 1939 paper that compared a variety of antennas for the emerging field of "high-definition" television. Carter showed conclusively that conical antennas held distinct advantages over dipoles and folded dipoles when it comes to broadband performance. Today, conical antennas are making a comeback for broadband applications such as digital television and UWB (ultra-wideband) or impulse radio. Stacked arrays of bowties and biconical dipoles are gradually displacing traditional mainstay antennas such as Yagis and log-periodics for the rooftop reception of digital television (DTV). One conical antenna, long popular among scanner hobbyists, the discone, has been described in previous articles in *Amateur Radio* magazines and books. The story has never been told fully, however. This article explains the history and theory of the discone, corrects some common misunderstandings, and presents an *EZNEC* model for a 0.6-octave discone that readers may copy and scale to their favorite frequency bands.

Conical antennas, and the discone in particular, have an obscure but fascinating history. Sergei Alexander Schelkunoff, at Bell Labs, was a titan of antenna theory in the early to mid 20th Century. In 1941, Schelkunoff published a major paper in the *Proceedings of the IRE*, which, among other things, analyzed the symmetric biconical dipole and showed that many other antennas can be analyzed

as extensions of it.<sup>1</sup> The discone antenna (Figure 1) is one such extension, in which the biconical dipole is asymmetric, one cone's angle being 90°, which gives a flat disk of radius equal to the cone length. Two years later, in 1943, Armin Kandorian at the Federal Telephone and Radio Corporation applied for a patent on the discone antenna. Kandorian's novel or inventive element was apparently that the antenna could be encased in a radome, making it suitable for aircraft, not that it used a cone or disk per se, those ideas being obvious in view of Schelkunoff's prior work. The patent was granted in 1945, whereupon Kandorian and his colleagues, Sichak, Felsenfeld, and Nail, at the newly renamed Federal Telecommunica-

<sup>1</sup>Notes appear on page 43.

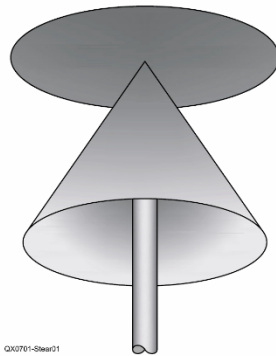
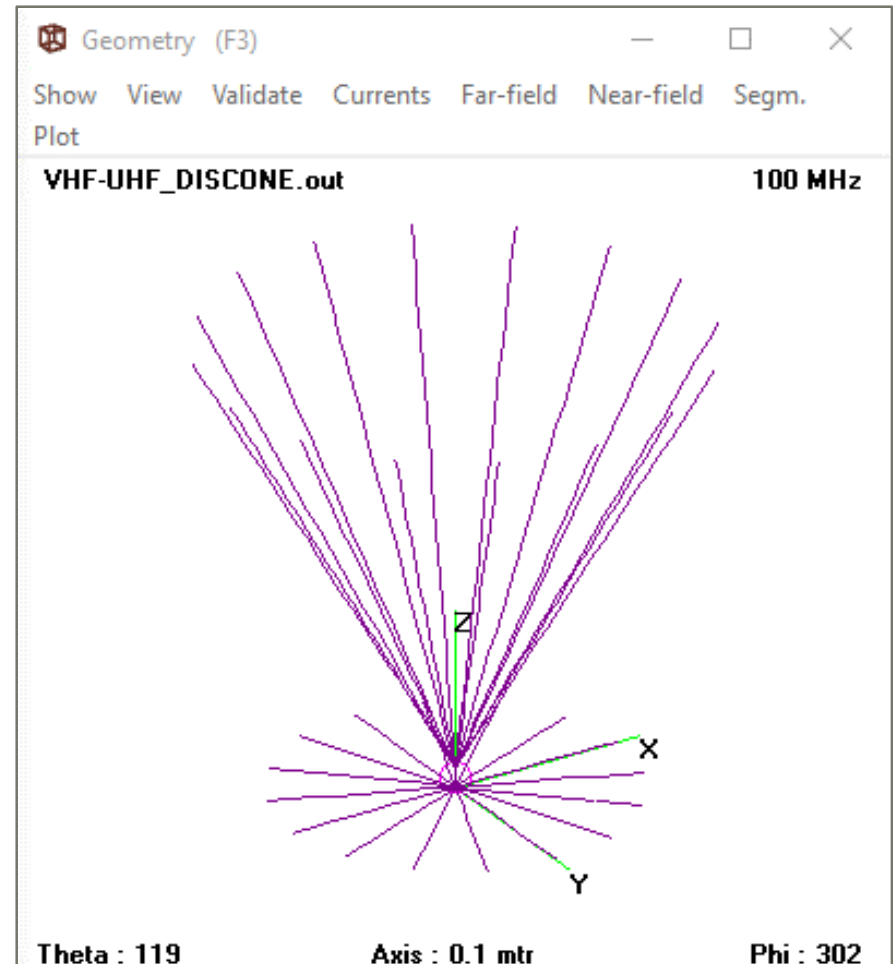


Figure 1 — This illustration shows a home-made discone for 2.4-GHz WiFi use. See [www.spaziolink.com/wifi](http://www.spaziolink.com/wifi).

QX0701-Steve01

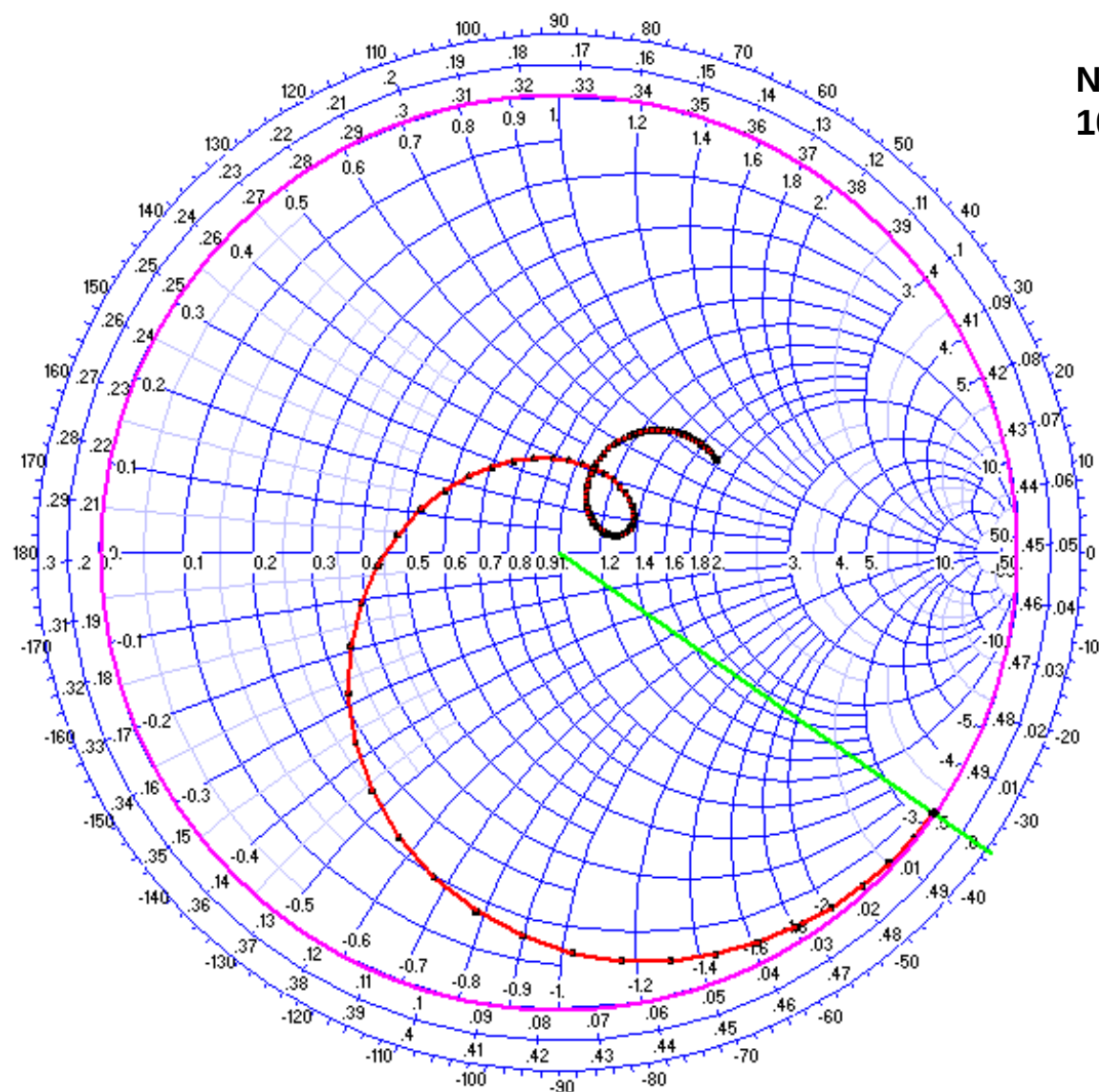
PO Box 4917  
Mountain View, CA 94040-0917  
k6oik@arrl.net

QEX Jan/Feb 2007 37



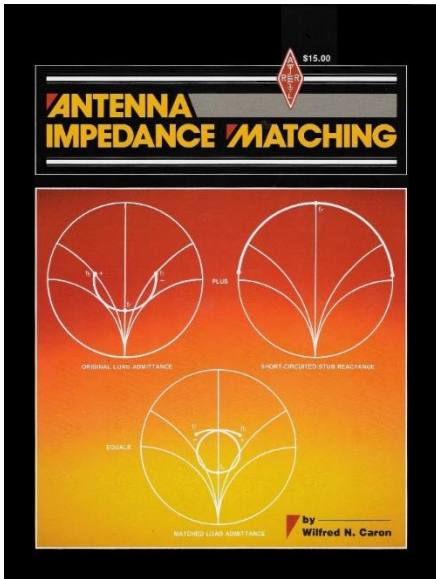
Steve Stearns, K6OIK "All About the Discone Antenna," QEX, pp. 37-44, January/February 2007.

## Example 5: VHF-UHF Discone Antenna

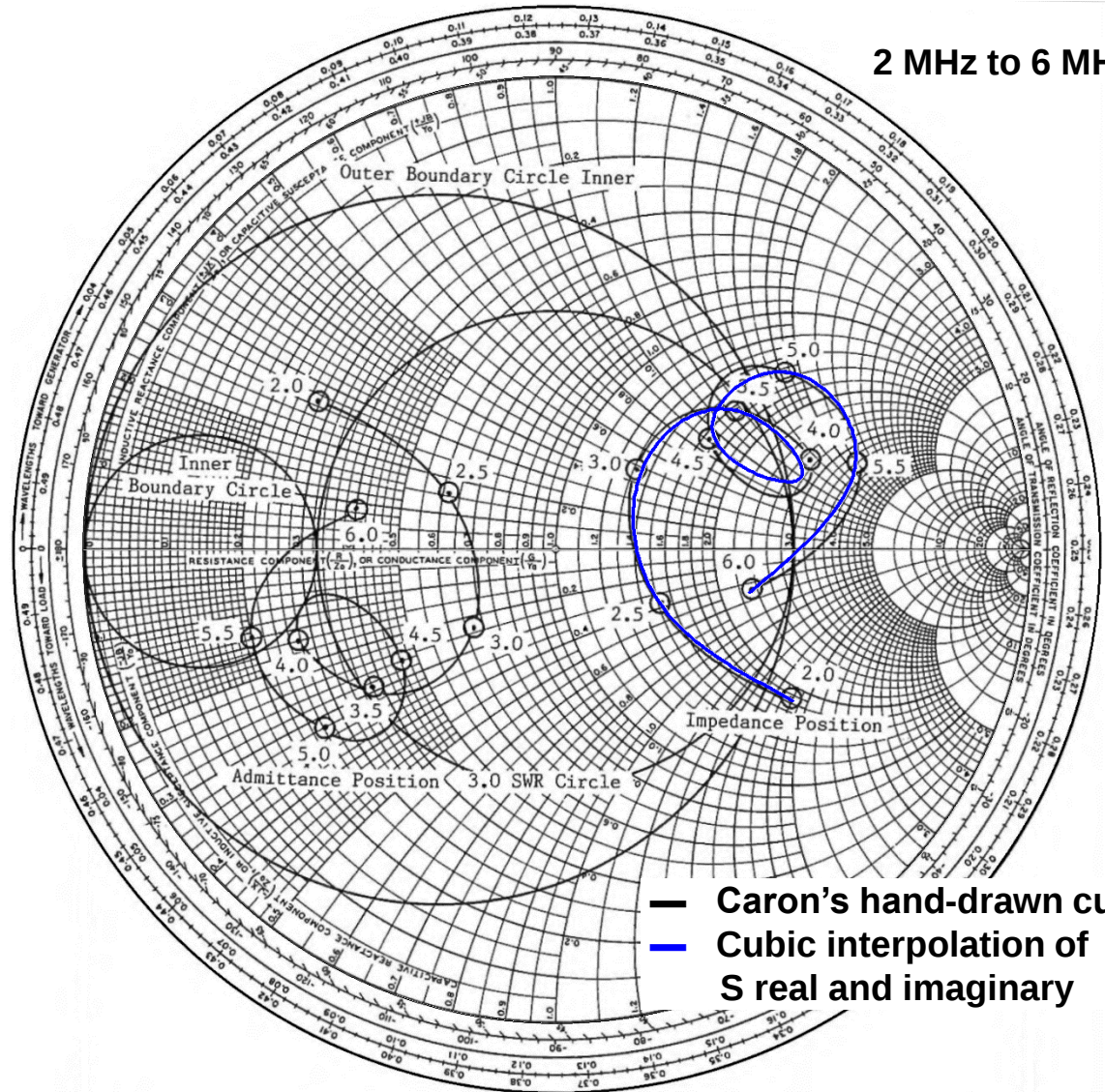


NEC2 Data  
100 MHz to 1 GHz

### Example 6: Broadband Dipole Antenna



**Wilfred N. Caron, *Antenna Impedance Matching*, ARRL, 1989**



## 2 MHz to 6 MHz

- Caron's hand-drawn curve
- Cubic interpolation of S real and imaginary



# Summary

---

- **Impedance curves bend or spiral clockwise as frequency increases**
  - A property of passive impedance functions, unrelated to Foster's Reactance theorem
- **Crossings of the horizontal axis of the Smith chart are called *resonances***
  - Downward crossings are called *anti-resonances*
- **An antenna can have any number of resonances**
  - An antenna can have ***no resonances***. Its impedance curve can lie entirely in the top or bottom half of the Smith chart
  - An antenna can have a ***finite*** number of resonances, e.g. fat dipoles, loop antennas, and bent blade antennas
  - An antenna can have an ***infinite*** number of resonances, e.g. dipole reactance given by the induced emf method

---

# **Impedance Model or Equivalent Circuit ?**

# Impedance Model vs Equivalent Circuit

$$R_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\gamma + 2\ln(2kl) - 2\text{Ci}(2kl) + \sin(2kl) [\text{Si}(4kl) - 2\text{Si}(2kl)] + \cos(2kl) [\text{Ci}(4kl) - 2\text{Ci}(2kl) + \gamma + \ln(kl)] \right\}$$
$$X_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\text{Si}(2kl) - \cos(2kl) [\text{Si}(4kl) - 2\text{Si}(2kl)] + \sin(2kl) [\text{Ci}(4kl) - 2\text{Ci}(2kl) + \gamma + 2\ln(ka) - \ln(kl)] \right\}$$

## ■ Impedance Models

- A mathematical formula or algorithm for calculating impedance
- Need not represent any physical device
- Can be any kind of function from simple to special
  - Polynomials
  - Rational functions
  - Sine and cosine integrals
  - Ad hoc formulas
- A compact compressed representation of impedance behavior
- Formula approximates given impedance behavior to a specified accuracy over a specified bandwidth
- Good for computation, interpolation, and SWR prediction, but not physical testing and measurement

## ■ Equivalent Circuits

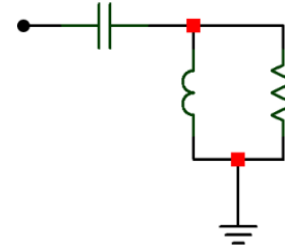
- A special kind of impedance model – an electric circuit or network
- Physically realizable
- Made of *passive* elements
  - Resistors
  - Inductors
  - Capacitors
  - Ideal transformers
- Impedance specified by a *positive real* rational function
- Circuit approximates given impedance behavior to a specified accuracy over a specified bandwidth
- Can build to make antenna dummies and emulators for reflection and transmission tests and measurements

# Impedance Model vs Equivalent Circuit

$$R_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\gamma + 2\ln(2kl) - 2\text{Ci}(2kl) + \sin(2kl) \right\}$$

$$X_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\text{Si}(2kl) - \cos(2kl) [ \text{Si}(4kl) - 2\text{Si}(2kl) ] \right\}$$

...



## ■ Impedance Models

- A mathematical formula or algorithm for calculating impedance
- Need not represent any physical device
- Can be any kind of function from simple to special
  - Polynomials
  - Rational functions
  - Sine and cosine integrals
  - Ad hoc formulas
- A compact compressed representation of impedance behavior
- Formula approximates given impedance behavior to a specified accuracy over a specified bandwidth
- Good for computation, interpolation, and SWR prediction, but not physical testing and measurement

## ■ Equivalent Circuits

- A special kind of impedance model – an electric circuit or network
- Physically realizable
- Made of *passive* elements
  - Resistors
  - Inductors
  - Capacitors
  - Ideal transformers
- Impedance specified by a *positive real* rational function
- Circuit approximates given impedance behavior to a specified accuracy over a specified bandwidth
- Can build to make antenna dummies and emulators for reflection and transmission tests and measurements

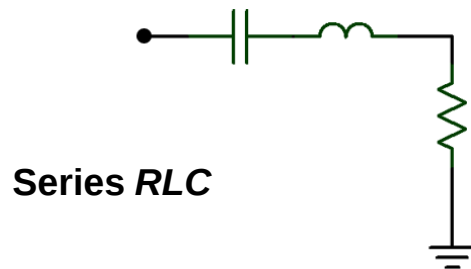
---

# **Low-Order Equivalent Circuits**

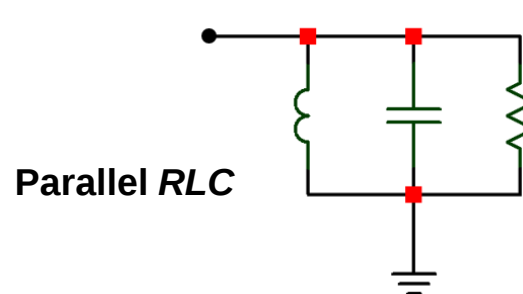
## **For narrowband modeling**



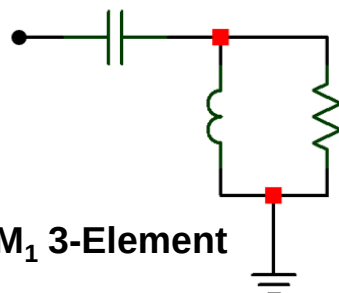
# Low-Order Equivalent Circuits for Narrow Bands Near 1<sup>st</sup> Resonance



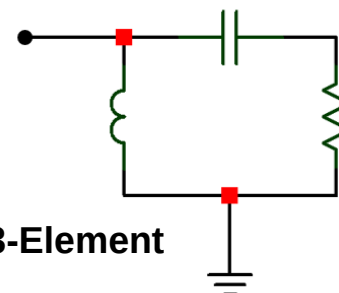
Series *RLC*



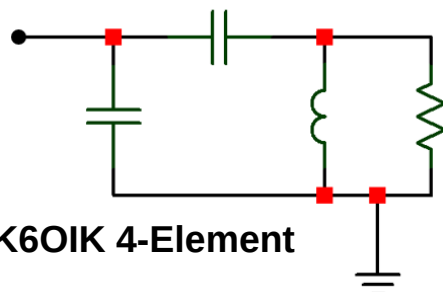
Parallel *RLC*



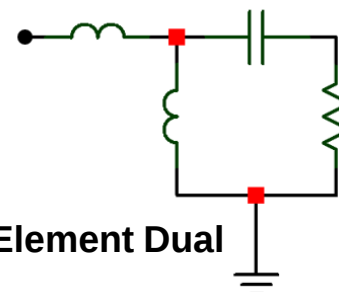
Chu TM<sub>1</sub> 3-Element



Chu TE<sub>1</sub> 3-Element



K6OIK 4-Element



K6OIK 4-Element Dual

Circuits for  
antennas near  
resonance

Circuits for  
antennas near  
antiresonance

# Series and Parallel RLC Equivalent Circuits

- **As an antenna impedance curve spirals around the Smith chart, there may be points of tangency with constant conductance circles or constant resistance circles**
  - “Tangency” here means location and direction, i.e. curvature sense
- **Tangencies define 3-element RLC equivalent circuits**
  - Resistance circle tangency  $\Rightarrow \frac{dR}{df} = 0$  and series RLC circuit
  - Conductance circle tangency  $\Rightarrow \frac{dG}{df} = 0$  and parallel RLC circuit
- **An antenna can have many narrowband RLC equivalent circuits at different frequencies, one for each tangency**

The resonant frequencies of such equivalent circuits don't equal, or imply the existence of, antenna resonant frequencies!

---

# **Defective Equivalent Circuits**

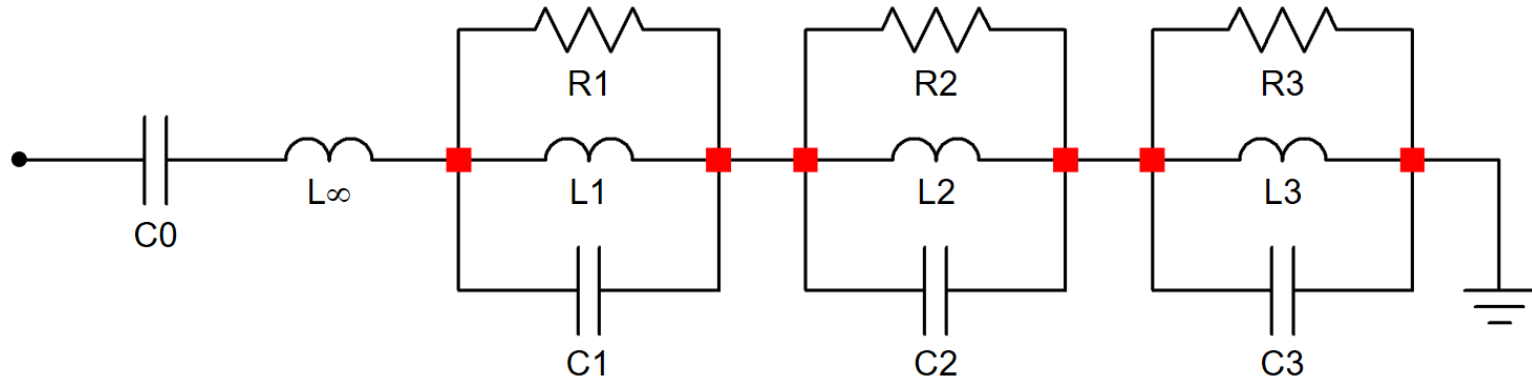
**Models vs equivalent circuits**

**Ad hoc**

**Unrealizable**

**Problematic stability**

# Foster's 1<sup>st</sup> Form, Modified for Small Losses



$$C_0 = 43.9 \text{ pF}$$

$$L_\infty = 4.49 \text{ mH}$$

$$C_1 = 22.9 \text{ pF}$$

$$L_1 = 12.5 \text{ mH}$$

$$R_1 = 4,970 \text{ } \Omega$$

$$C_2 = 30.3 \text{ pF}$$

$$L_2 = 2.26 \text{ mH}$$

$$R_2 = 3,338 \text{ } \Omega$$

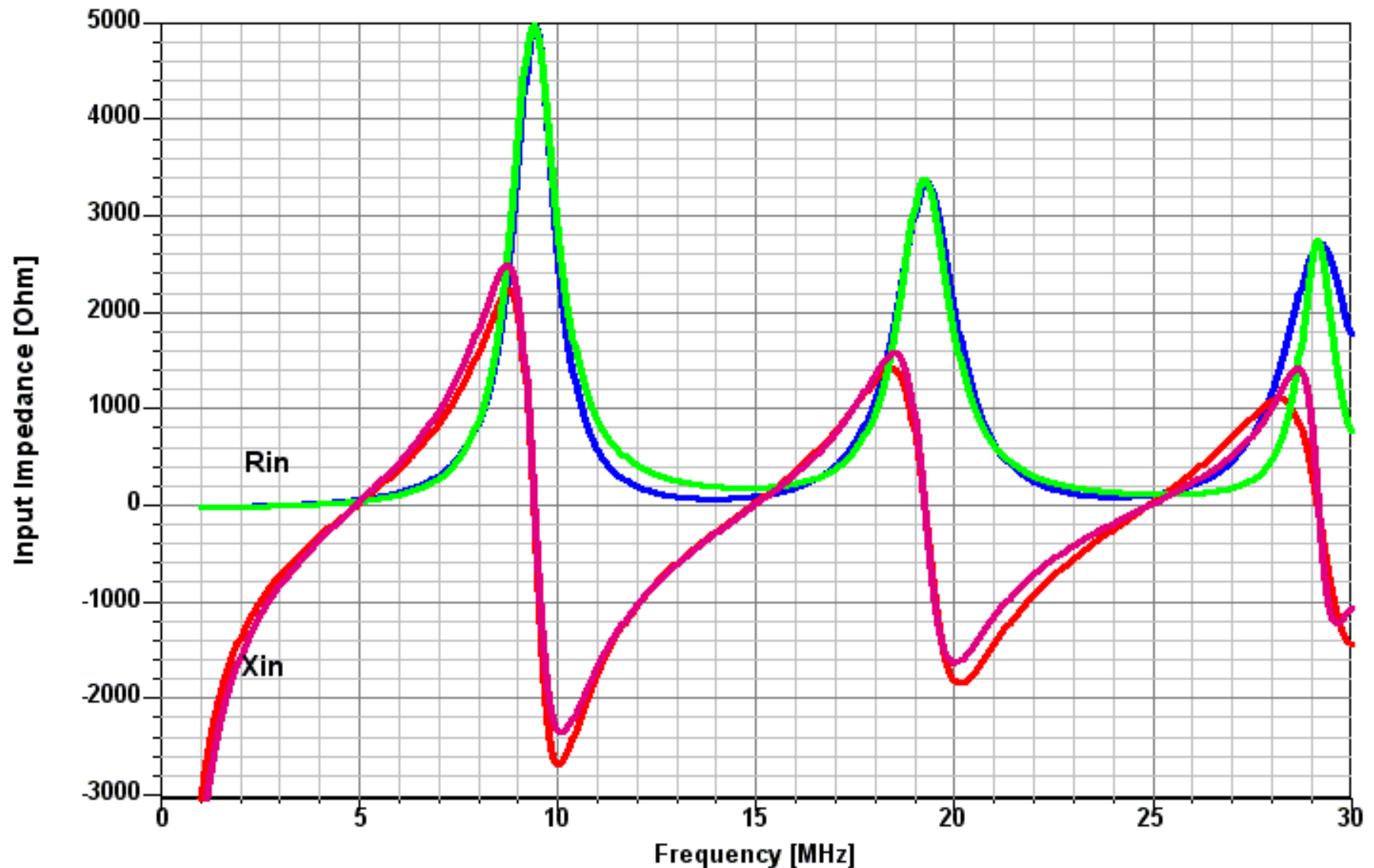
$$C_3 = 57.1 \text{ pF}$$

$$L_3 = 522 \text{ nH}$$

$$R_3 = 2,702 \text{ } \Omega$$

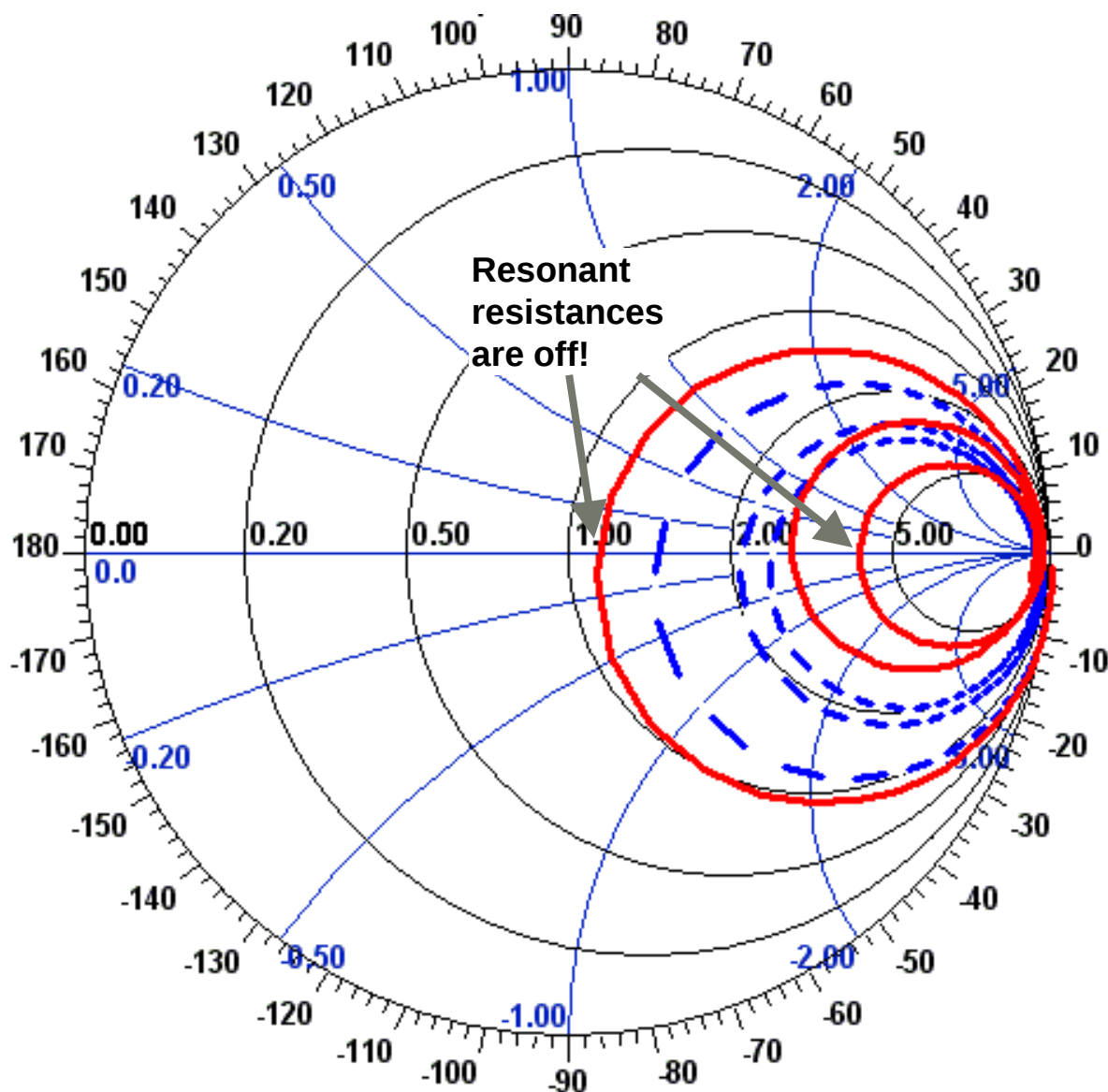
- **Ramo, Whinnery, and Van Duzer, *Fields and Waves in Communication Electronics*, Wiley, 1965. Section 11.13**
  - Tang-Tieng-Gunn (1993)
  - Hamid-Hamid (1997)
  - Rambabu-Ramesh-Kalghatgi (1999)
- **Fits dipole impedance better near antiresonances, worse near resonances**

# Accuracy of Hamid & Hamid's Equivalent Circuit

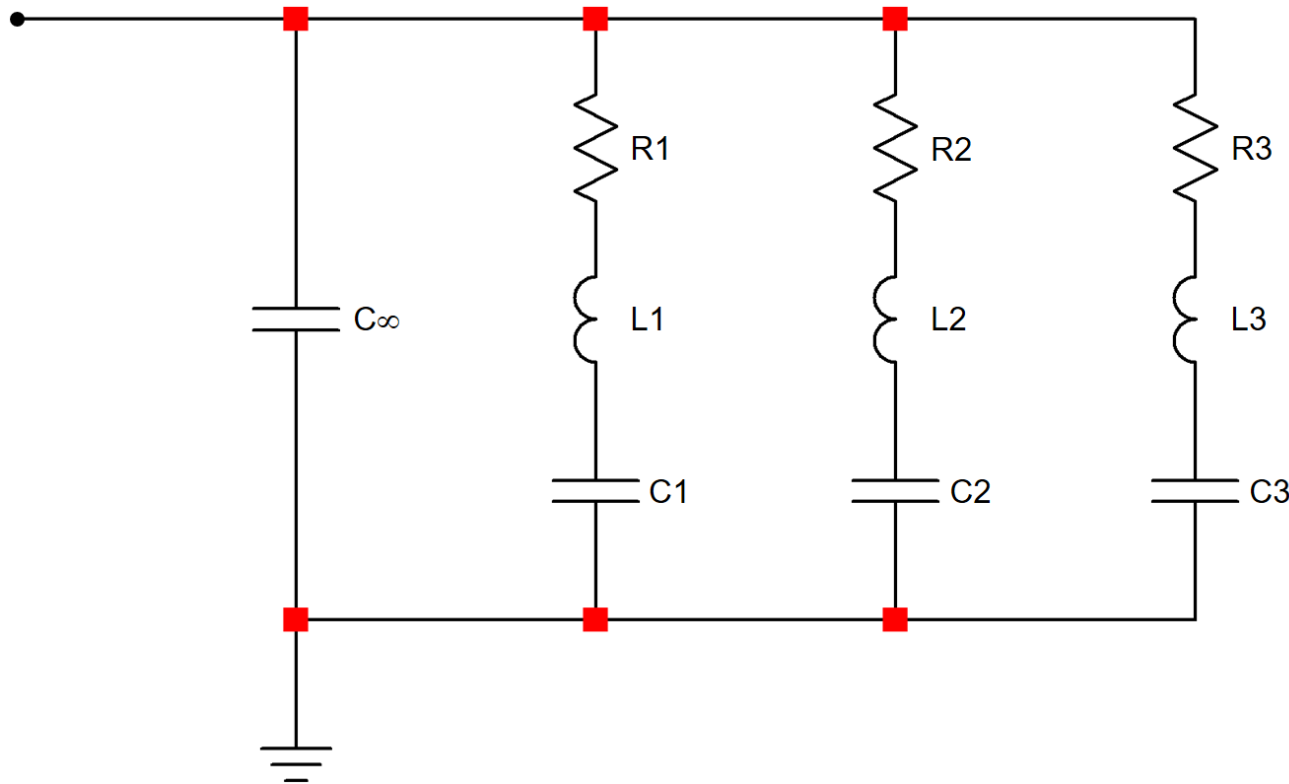




# Accuracy of Hamid & Hamid's Equivalent Circuit



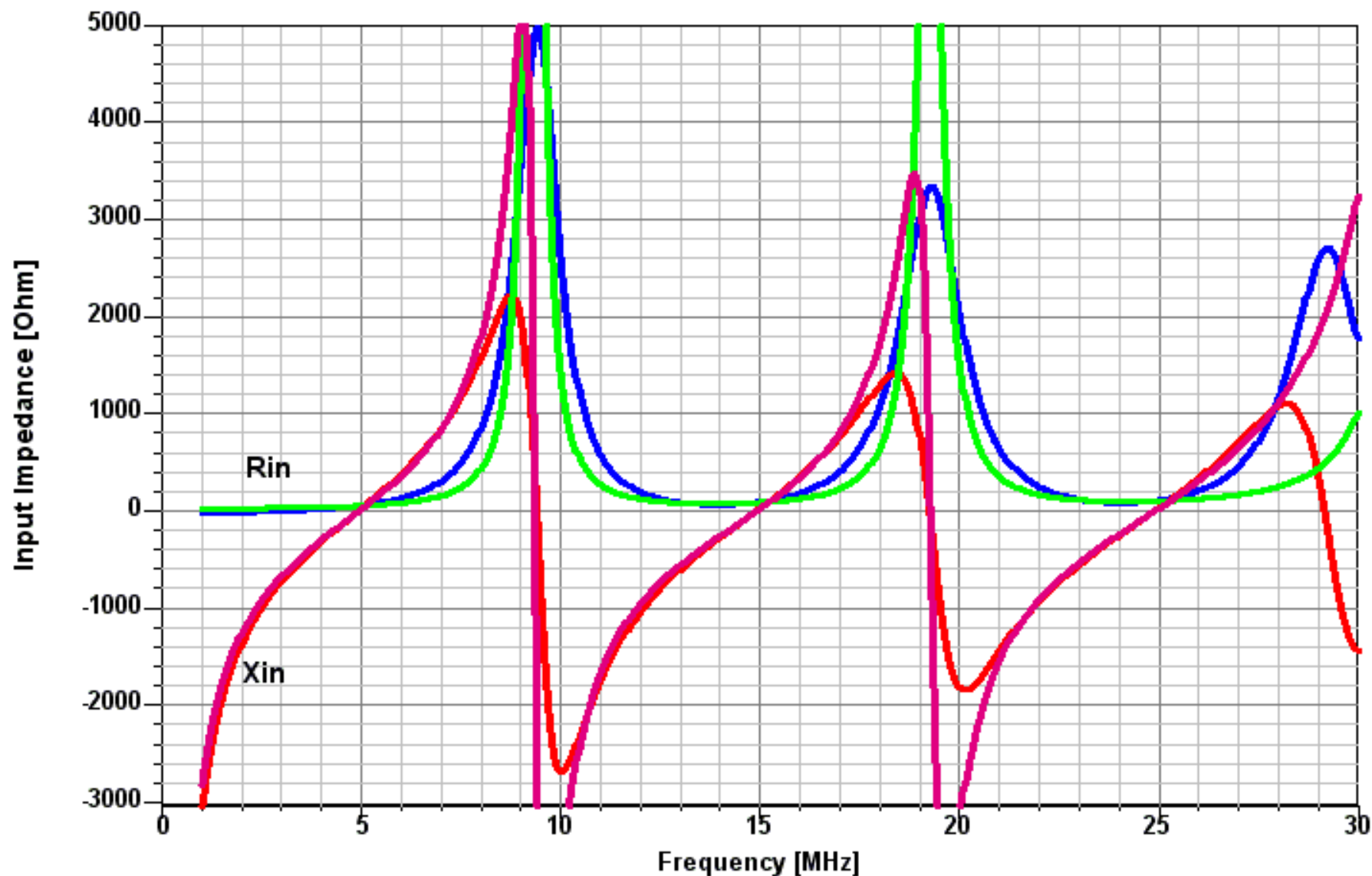
# Foster's 2<sup>nd</sup> Form, Modified for Small Losses



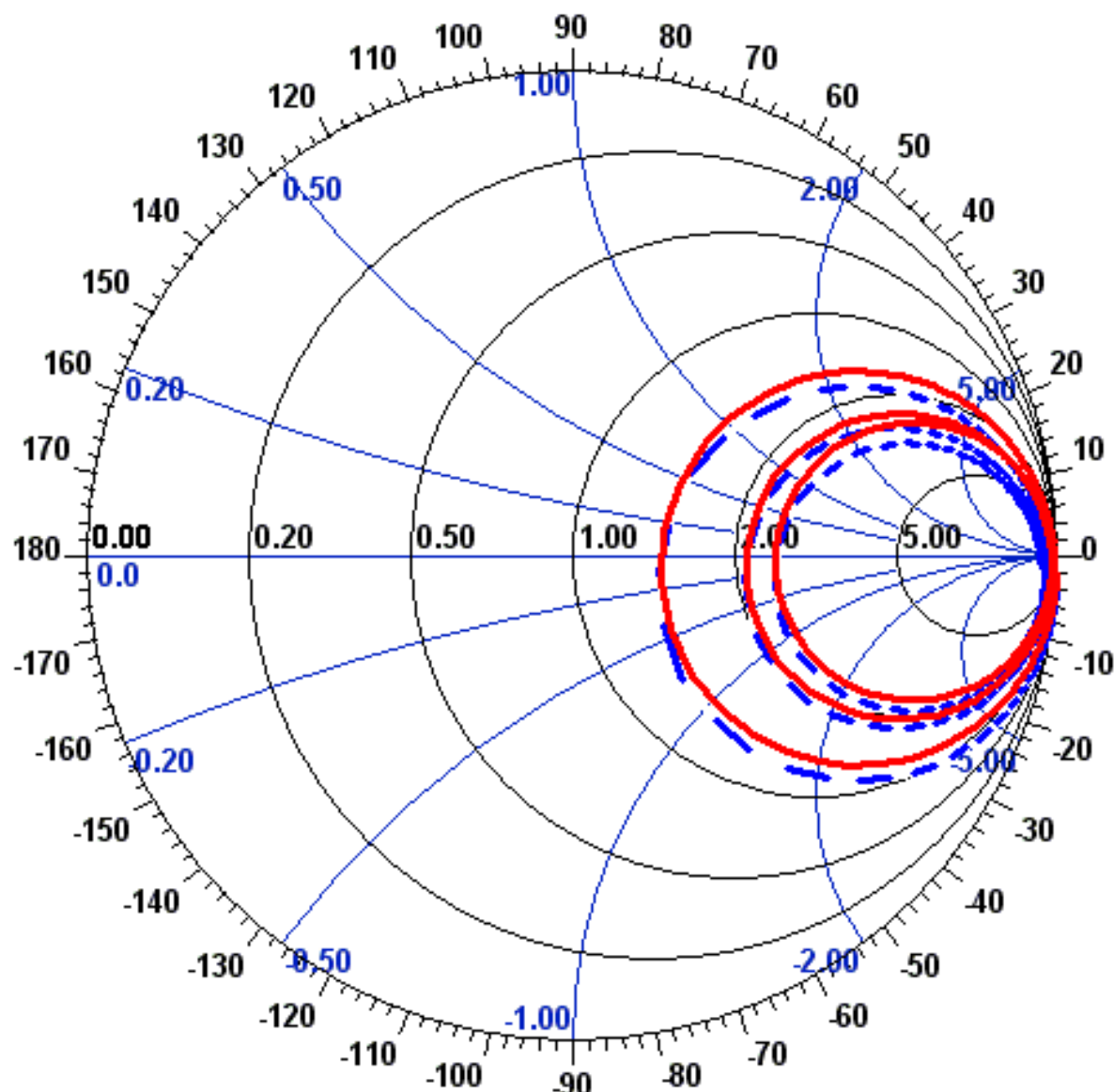
$C_\infty = 5.44 \text{ pF}$   
 $C_1 = 42.9 \text{ pF}$   
 $C_2 = 5.05 \text{ pF}$   
 $C_3 = 1.92 \text{ pF}$   
 $L_0 = \infty$   
 $L_1 = 24.9 \text{ mH}$   
 $L_2 = 22.8 \text{ mH}$   
 $L_3 = 21.4 \text{ mH}$   
 $R_1 = 72.2 \text{ } \Omega$   
 $R_2 = 106 \text{ } \Omega$   
 $R_3 = 122 \text{ } \Omega$

- Fits dipole impedance better near resonances, worse near antiresonances
- McKinley *et al.* (2012) used similar topology for circular loop antennas

# Accuracy of Foster's 2<sup>nd</sup> Form With Small Losses

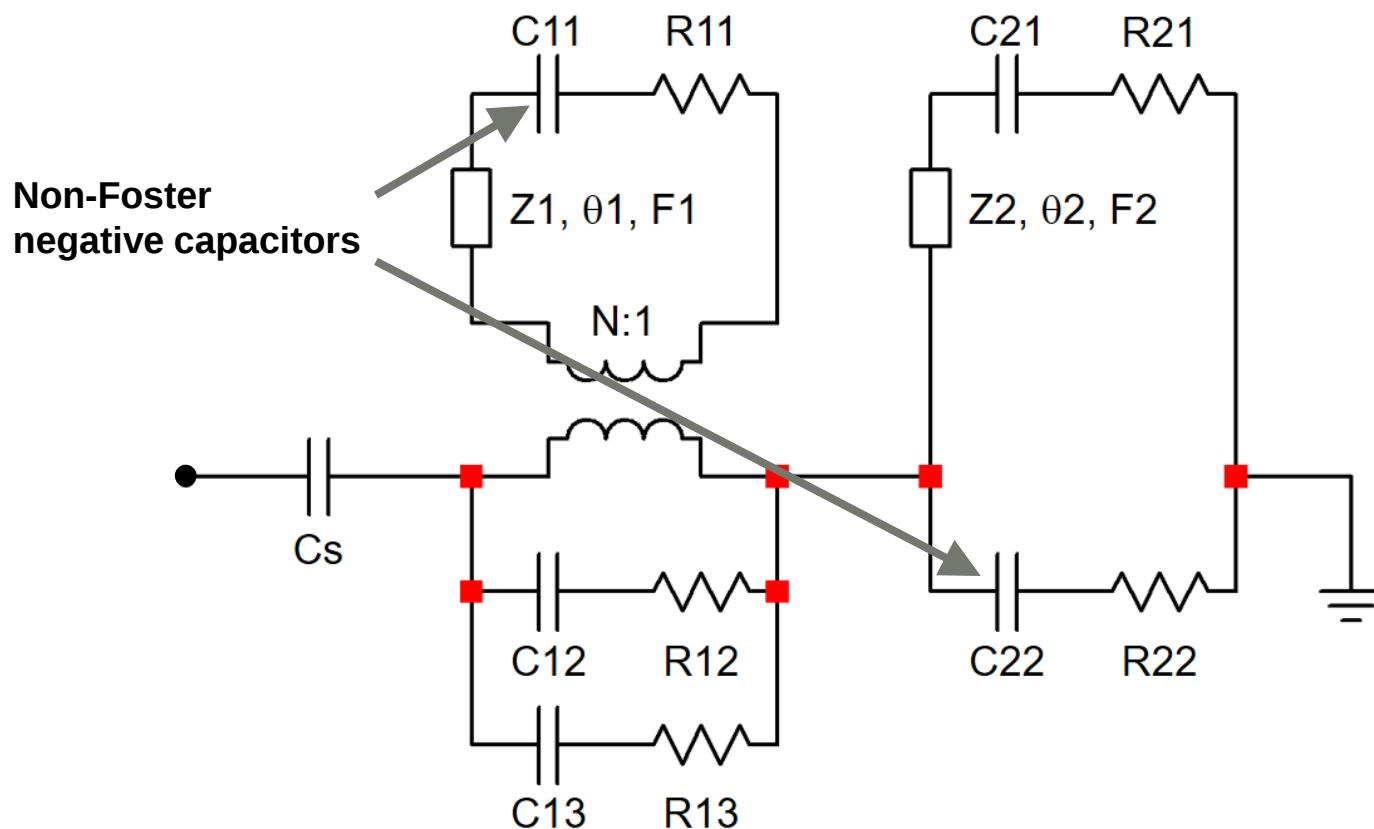


# Accuracy of Foster's 2<sup>nd</sup> Form With Small Losses



# Long, Werner, & Werner's Broadband Model (2000)

## Frequency Scaled to $f_0 = 5 \text{ MHz}$ , $\Omega' = 7.8$



$$C_s = 150 \text{ pF}$$

$$Z_1 = 215 \Omega$$

$$Z_2 = 195 \Omega$$

$$N = 1$$

$$C_{11} = -975 \text{ pF}$$

$$\theta_1 = 44.9^\circ$$

$$C_{12} = 24.0 \text{ pF}$$

$$C_{13} = 8.33 \text{ pF}$$

$$R_{11} = 13.1 \Omega$$

$$F_1 = 5 \text{ MHz}$$

$$R_{12} = 3,600 \Omega$$

$$R_{13} = 500 \Omega$$

$$C_{21} = 17.6 \text{ pF}$$

$$\theta_2 = 46.9^\circ$$

$$C_{22} = -3.00 \text{ pF}$$

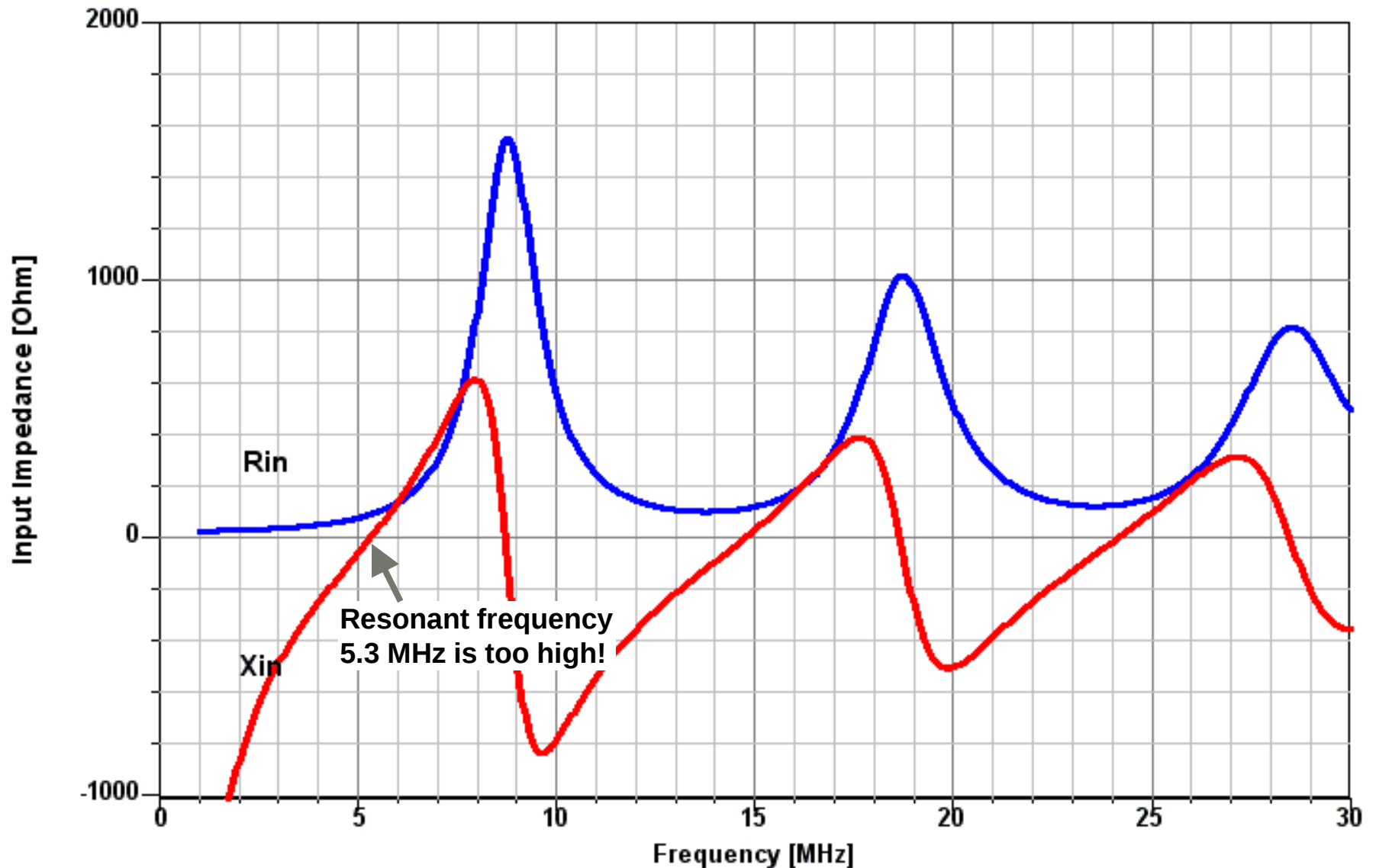
$$R_{21} = 700 \Omega$$

$$F_2 = 5 \text{ MHz}$$

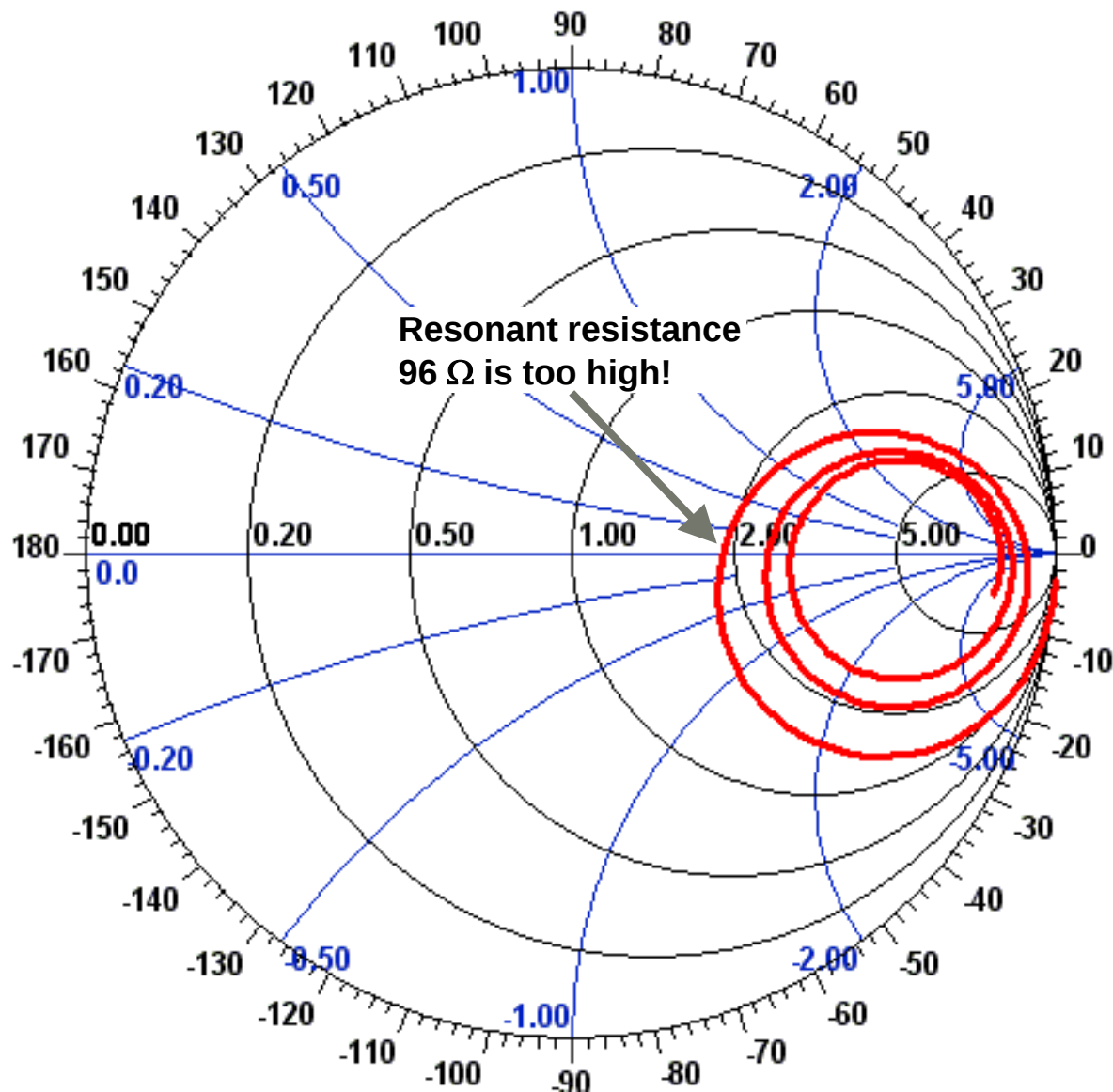
$$R_{22} = 295 \Omega$$



# Accuracy of Long, Werner, & Werner's Model

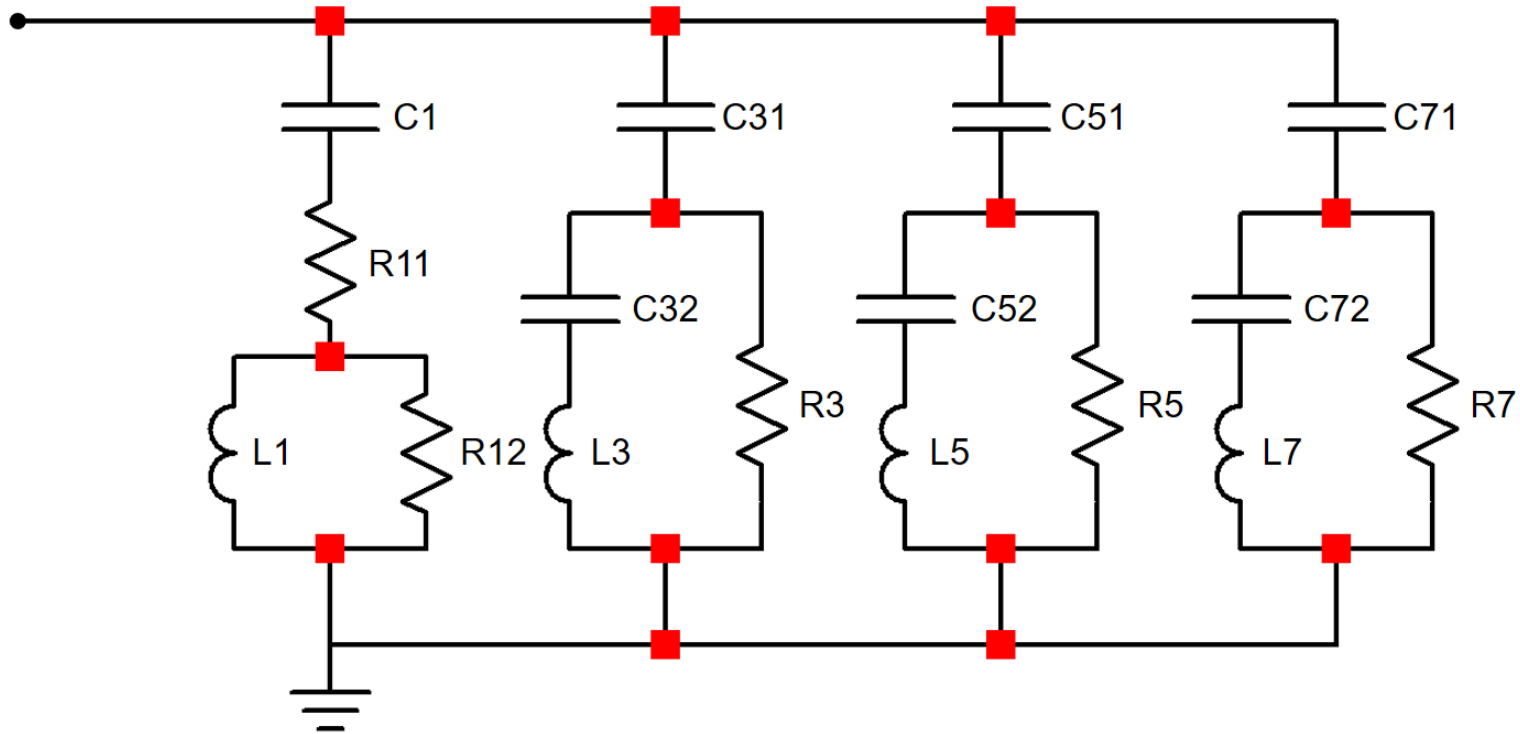


# Accuracy of Long, Werner, & Werner's Model



# Streable & Pearson's Broadband Equivalent Circuit

Frequency Scaled to  $f_0 = 5 \text{ MHz}$ ,  $\Omega' = 10.6$



C1 = 86.6 pF

L1 = 13.8 mH

R11 = 0.663  $\Omega$

R12 = 2,201  $\Omega$

C31 = 15.0 pF

C32 = 33.8 pF

L3 = 11.7 mH

R3 = 4,959  $\Omega$

C51 = 7.17 pF

C52 = 8.87 pF

L5 = 10.9 mH

R5 = 6,514  $\Omega$

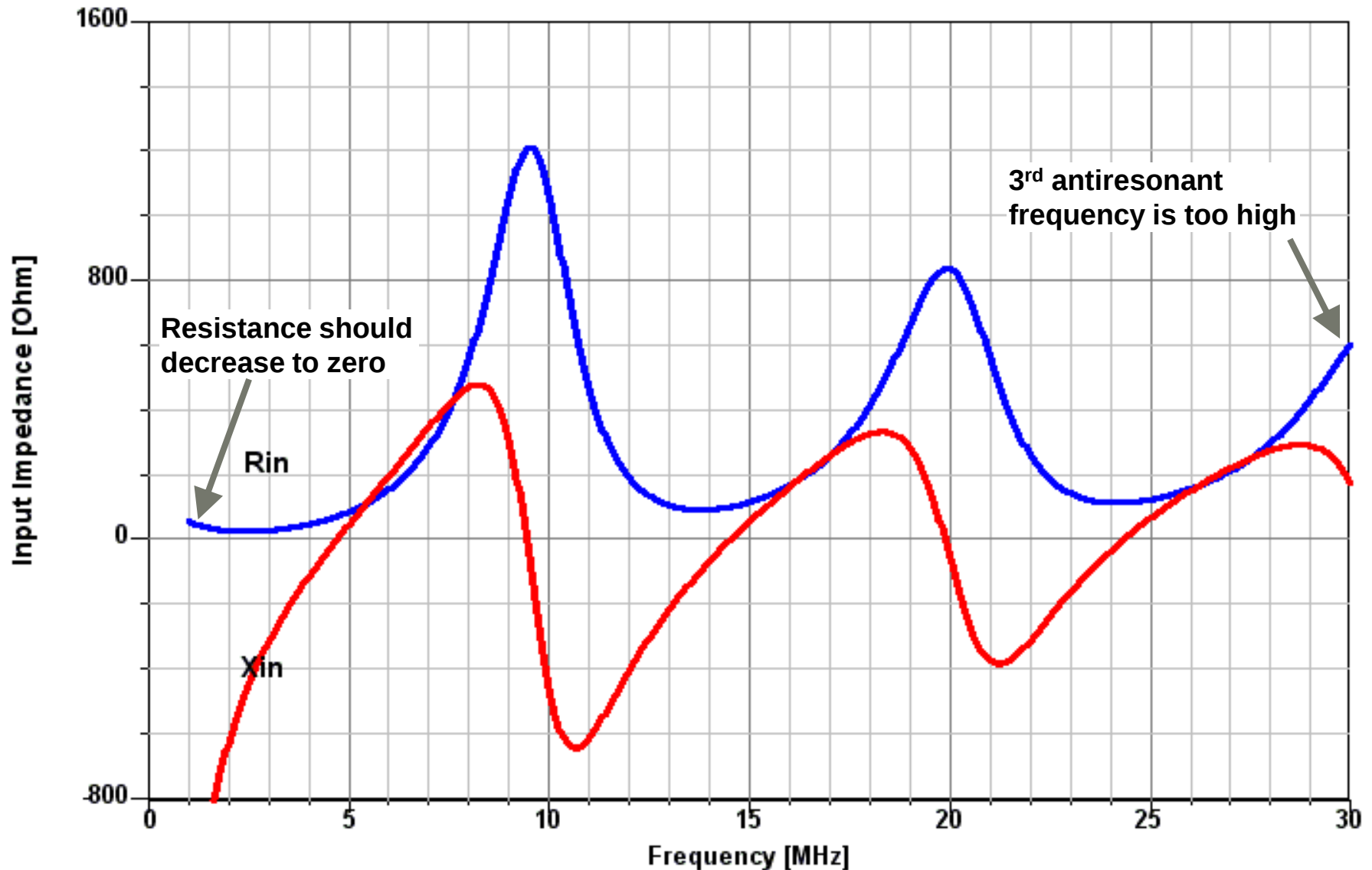
C71 = 4.51 pF

C72 = 3.98 pF

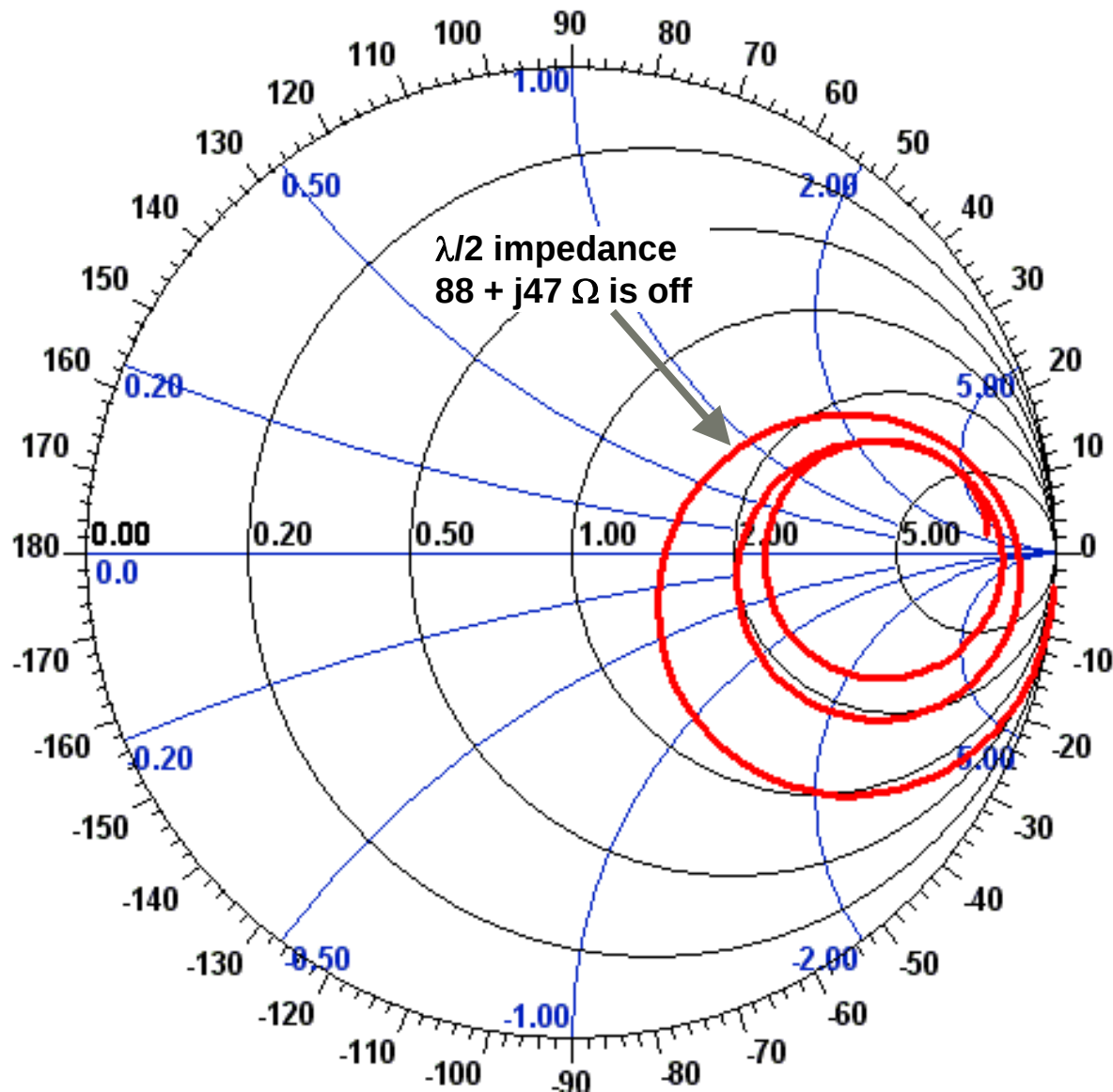
L7 = 10.3 mH

R7 = 7,542  $\Omega$

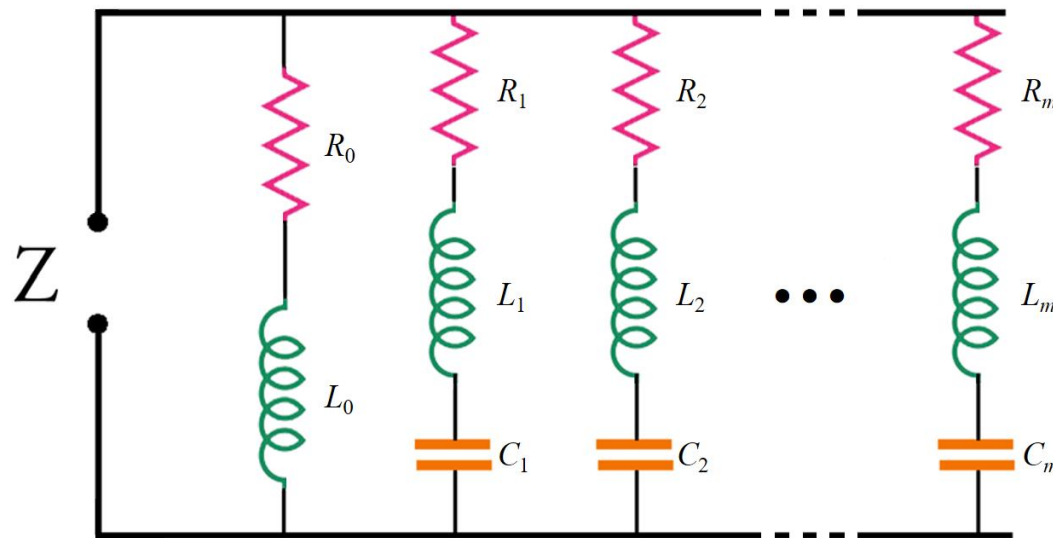
# Accuracy of Streable & Pearson's Equivalent Circuit



# Accuracy of Streable & Pearson's Equivalent Circuit



# McKinley *et al.* Circular Loop Antenna Model



- McKinley *et al.* (2012) broadband model for large circular loop antennas derived from analyses of Storer (1955) and Wu (1962)
- Circuit topology similar to Foster's 2<sup>nd</sup> form with loss
- Absence of capacitor in 1<sup>st</sup> stage puts admittance pole near origin and antiresonance occurs below resonance
- Model is unrealizable because elements are functions of frequency, not constants
- Assumed incorrectly that each current expansion function corresponds to a single system mode and natural frequency, thereby confusing expansion functions with modes
- Moreover, the modes of circular loop antennas were already solved by R.F. Blackburn (1976)

A.F. McKinley, *et al.*, "The Analytical Basis for the Resonances and Anti-Resonances of Loop Antennas and Meta-Material Ring Resonators," *J. Applied Physics*, vol. 112, no. 9, Nov. 2012.  
J.E. Storer, "Impedance of Thin-Wire Loop Antennas," *Transactions of the AIEE*, vol. 75, no. 5, pp. 606-619, Nov. 1956.  
T.T. Wu, Theory of the Thin Circular Loop Antenna, *J. of Mathematical Physics*, vol. 3, no. 6, pp. 1301-1304, Nov-Dec. 1962.  
R.F. Blackburn, *Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method*, Ph.D. dissertation, U. Mississippi, ADA033089, USAF, Nov. 1976.



# Problematic “Equivalent Circuits”

- **Antenna models that use non-passive elements, such as square-law resistors,  $R(f) = Rf^2$ , are active and therefore unrealizable and/or problematic with respect to stability**
  - Witt (1995)
  - Long, Werner, and Werner (2000)
  - Rudish and Sussman-Fort (2002)
  - Aberle and Romak (2007)
  - Karawas and Collin (2008)
  - McKinley, White, Maksymov, and Catchpole (2012)
- **Such models are mathematical representations of impedance but should not be called “equivalent circuits”**
- **The term “equivalent circuit” is reserved for passive, realizable circuit models of impedance**

# Comparison of Antenna Equivalent Circuits

## K6OIK, Pacificon 2003, 2007

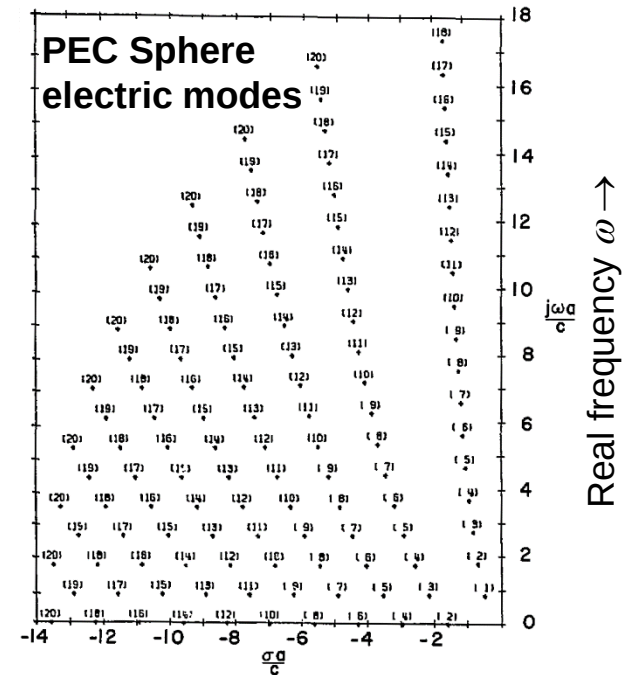
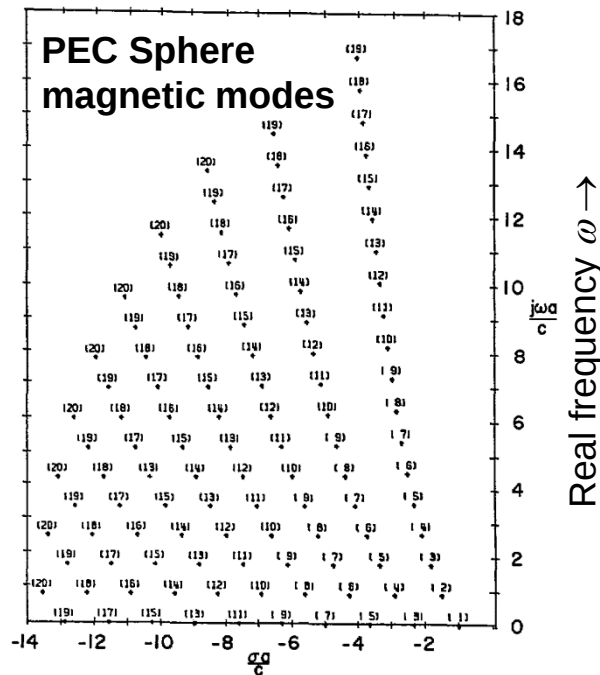
| Antenna Impedance Model                        | Approximation Accuracy         | Realizable Equivalent Circuit | Darlington Form | Element Types      | Maximum Frequency Range  |
|------------------------------------------------|--------------------------------|-------------------------------|-----------------|--------------------|--------------------------|
| Series RLC                                     | fair                           | yes                           | yes             | $R, L, C$          | $0.94 f_0$ to $1.05 f_0$ |
| Witt model                                     | fair                           | no                            | no              | $R(f)$ and TL stub | $0.6 f_0$ to $1.2 f_0$   |
| Chu 3-Element                                  | good                           | yes                           | yes             | $R, L, C$          | $0.90 f_0$ to $1.08 f_0$ |
| Tang-Tien-Gunn 4-Element                       | excellent                      | yes                           | yes             | $R, L, C$          | DC to $1.4 f_0$          |
| Schelkunoff 4-Element                          | excellent                      | yes                           | yes             | $R, L, C, TL$      | DC to $1.4 f_0$          |
| Author's 5-Element                             | excellent                      | yes                           | yes             | $R, L, C$          | DC to $1.4 f_0$          |
| Fosters 1 <sup>st</sup> Form with small losses | poor, best near antiresonances | yes                           | no              | $R, L, C$          | no limit                 |
| Fosters 2 <sup>nd</sup> Form with small losses | poor, best near resonances     | yes                           | no              | $R, L, C$          | no limit                 |
| Long-Werner-Werner                             | fair                           | no                            | no              | $R, C, TL$         | 5 octaves                |
| Streable-Pearson                               | good                           | yes                           | no              | $R, L, C$          | no limit                 |
| Author's Broadband                             | excellent                      | yes                           | no              | $R, L, C$          | no limit                 |
| Schelkunoff TL cascade                         | fair                           | yes                           | yes             | $R, L, C, TL$      | limited                  |
| Spherical TE-TM modes                          | excellent                      | yes                           | no              | $R, L, C$          | no limit                 |

---

# **Universal Equivalent Circuits**

**For all antennas  
Over any bandwidth**

# Antennas Ring at Many Frequencies (Like Bells)



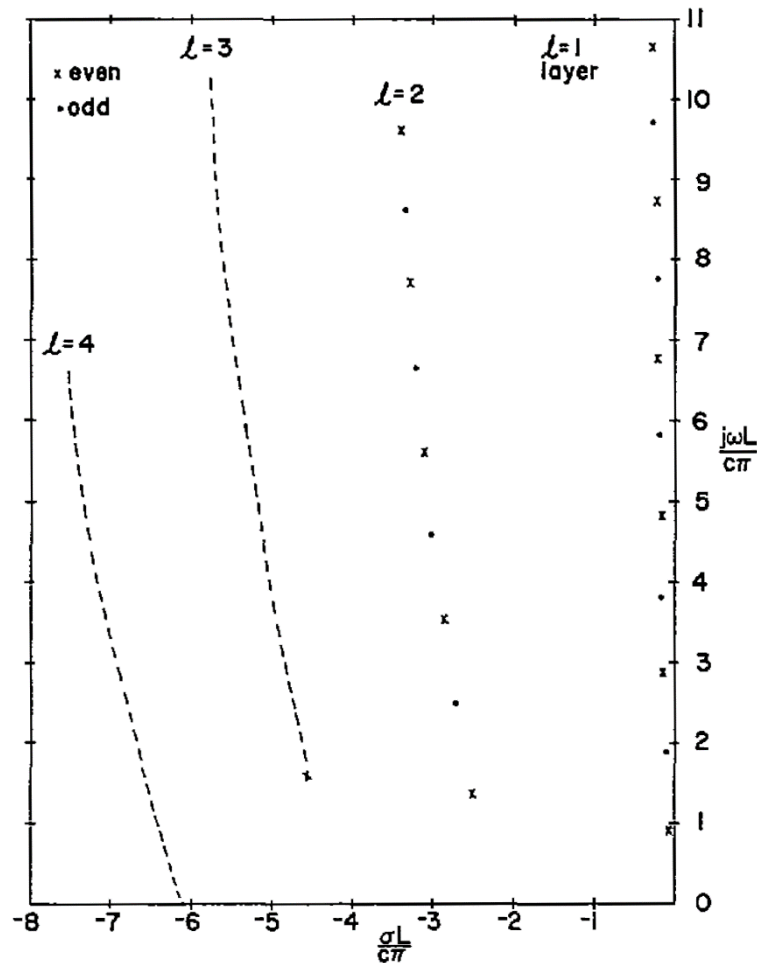
- **A bell's tone is not pure but consists of many tones (modes)**

- Initial roughness due to most partial tones damps out, leaving five audible "partial" tones
  - Hum: The lowest partial, one octave below Prime
  - Prime: The main partial, one octave above Hum
  - Tierce: A minor third above Prime
  - Quint: A perfect fifth above Prime
  - Nominal: One octave above Prime
- Late tone is pure

- **Antennas have natural frequencies (modes) like bells**

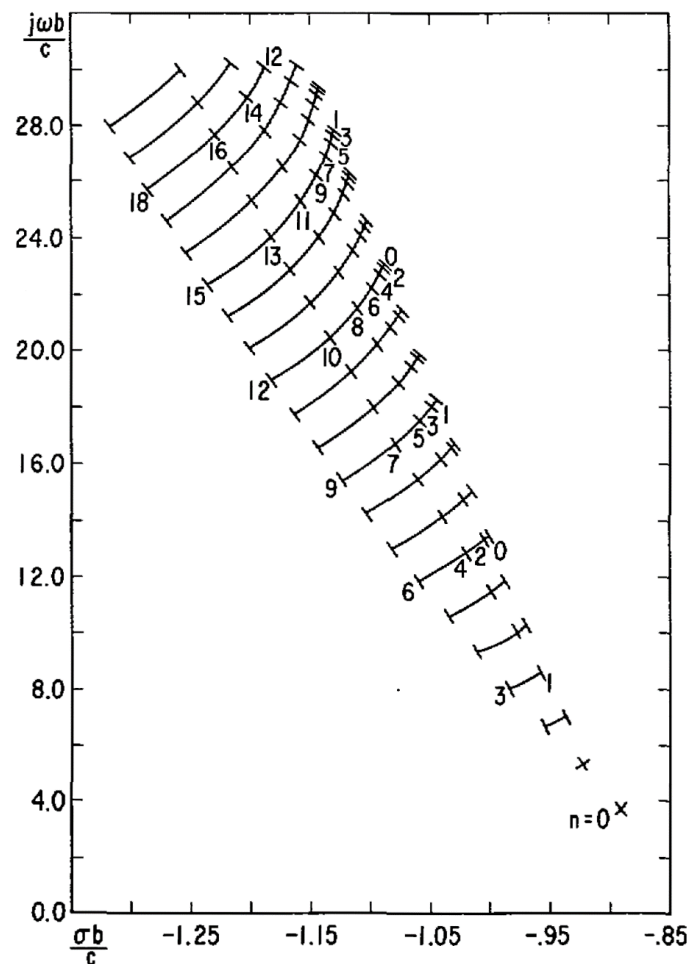
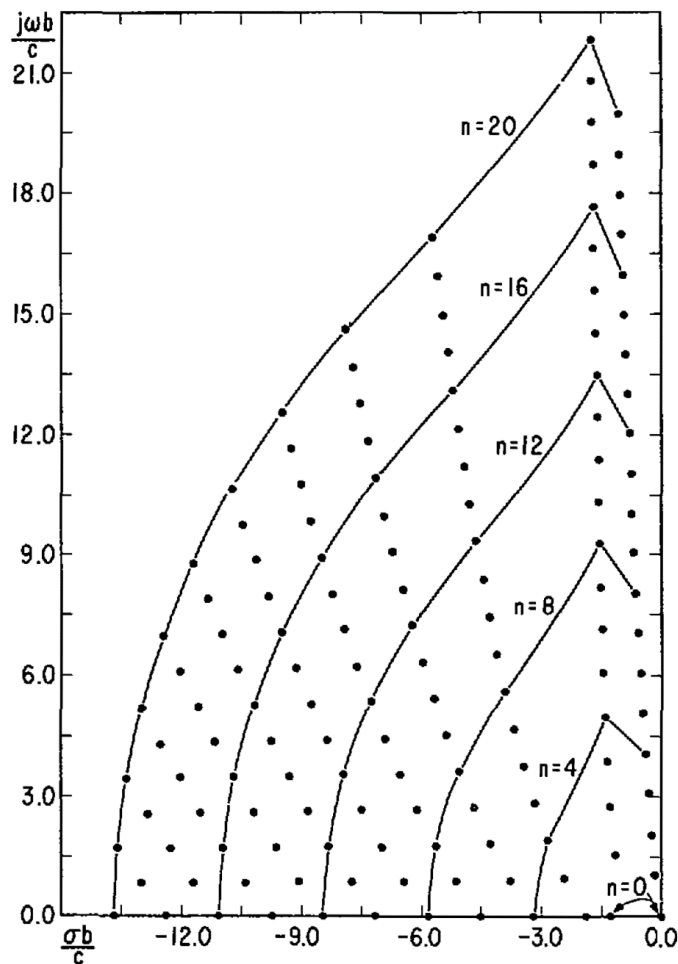
- Impedance resonances are determined by how modal currents sum

# Dipole and Monopole Natural Frequencies (Admittance Poles)



F.M. Tesche, "On the Analysis of Scattering and Antenna Problems Using the Singularity Expansion Technique," *IEEE Transactions on Antennas and Propagation*, vol. 21, no. 1, pp. 53-62, January 1973.

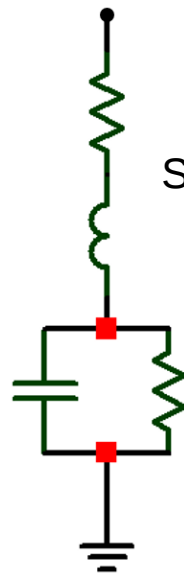
# Circular Loop Antenna Natural Frequencies (Admittance Poles)



R.F. Blackburn, *Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method*, Ph.D. dissertation, U. Mississippi, May 1976.  
 R.F. Blackburn and D.R. Wilton, "Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method," *IEEE Transactions on Antennas and Propagation*, vol. 26, no. 1, pp. 136-140, January 1978.

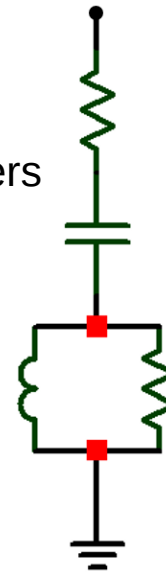


# Universal Equivalent Circuits – Four Kinds of Stages

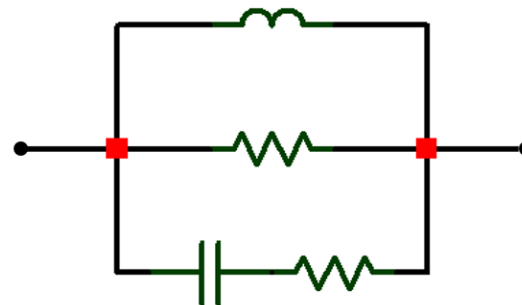
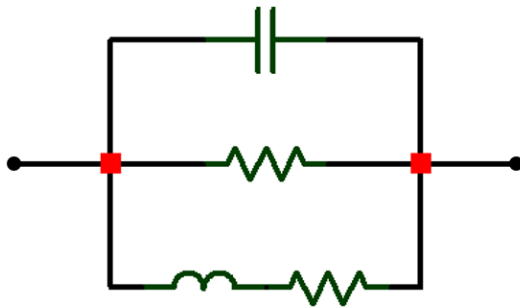


Stages for Type 2 parallel ladders

- Stages must have four degrees of freedom
- Needed to set complex pole location and complex residue



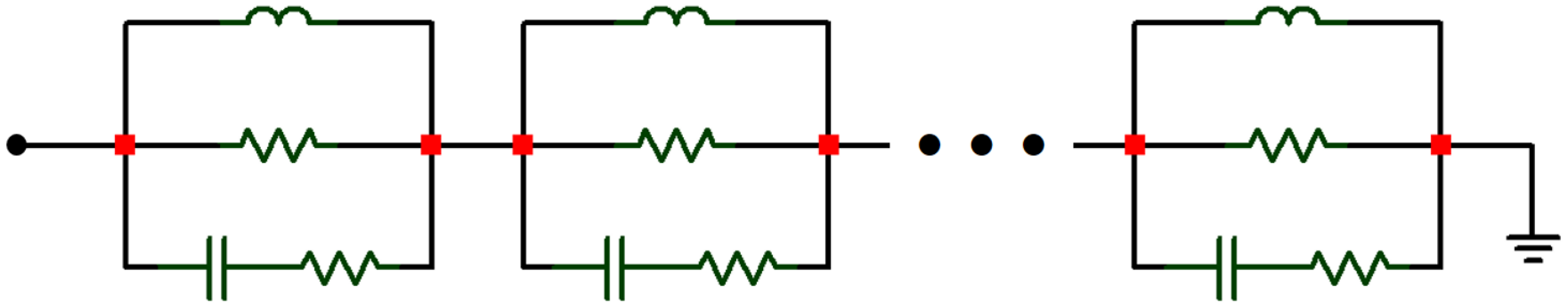
Stages for Type 1 series ladders



M. K. Zinn, "Network Representation of Transcendental Impedance Functions," *Bell System Technical Journal*, vol. 31, no. 2, pp. 378-404, March 1952.

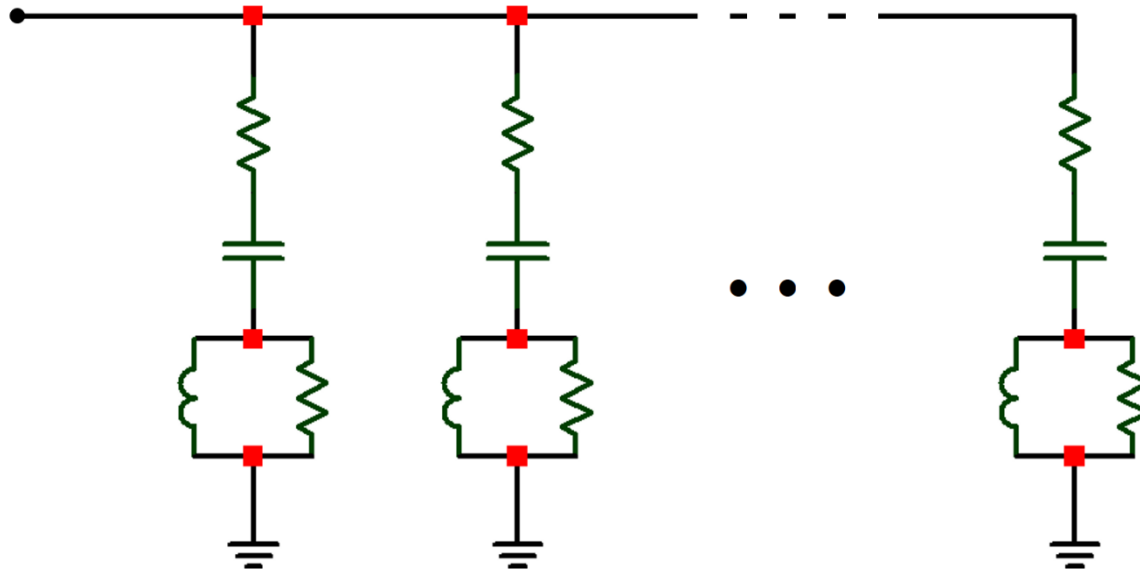
S. A. Schelkunoff, "Representation of Impedance Functions in Terms of Resonant Frequencies," *Proceedings of the IRE*, vol. 32, no. 2, pp. 83-90, February 1944.

# Universal Equivalent Circuit – Type 1



- Type 1 ladder based on impedance poles (admittance zeros)
- Similar to Foster's 1<sup>st</sup> form but with two resistors per stage
- Each stage gives a conjugate pair of poles of impedance
- Chu's TE<sub>1</sub> equivalent circuit is a special case (first stage)
- Circuit can be converted to single-resistor form by Darlington synthesis

# Universal Equivalent Circuit – Type 2

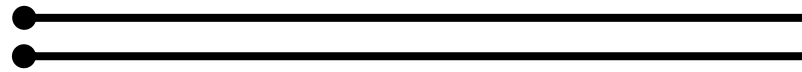


- Type 2 ladder based on admittance poles (impedance zeros)
- Similar to Foster's 2<sup>nd</sup> form but with two resistors per stage
- Each stage gives a conjugate pair of poles of admittance
- Chu's  $TM_1$  equivalent circuit is a special case (one stage)
- Circuit can be converted to single-resistor form by Darlington synthesis

---

# Examples

# Open Stub Equivalent Circuit

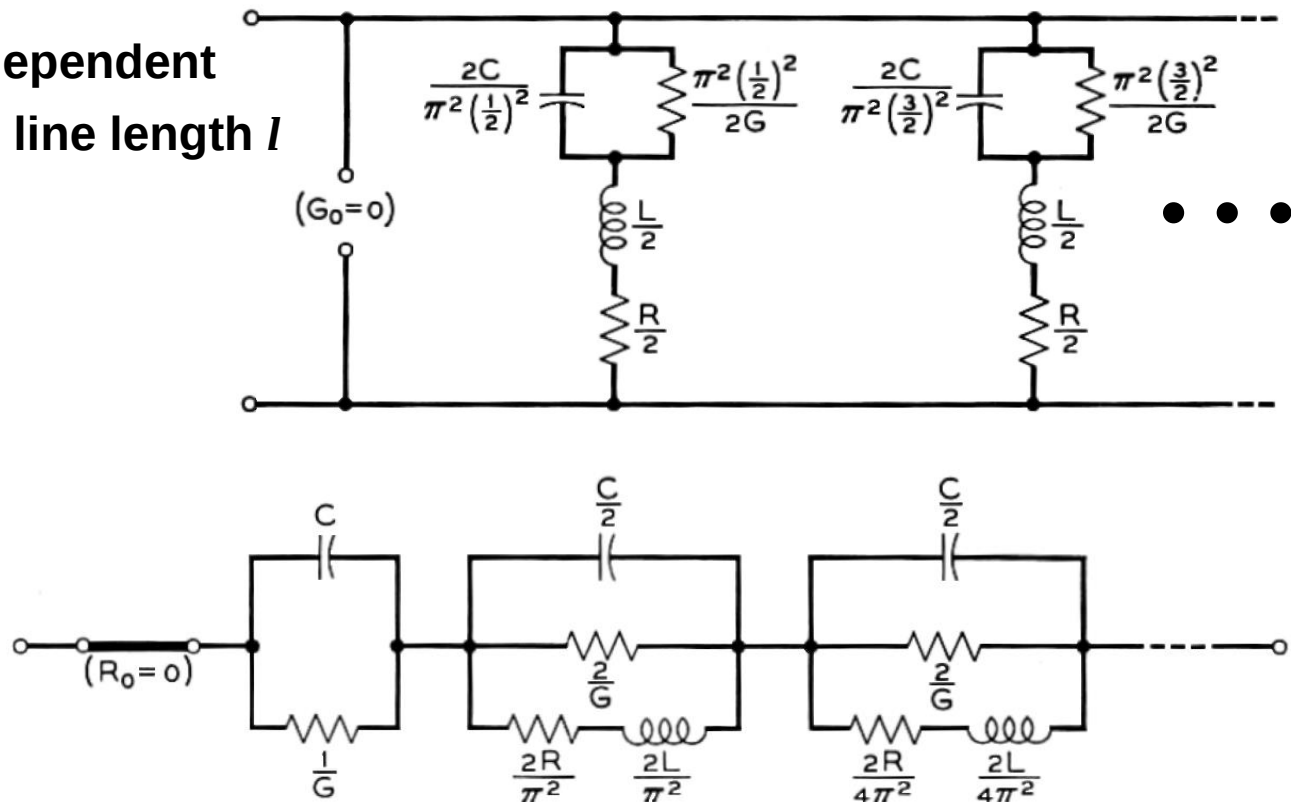


$$Z_{in} = Z_0 \coth \gamma l = Z_0 \coth(\alpha l + j\beta l)$$

- Loss is frequency-independent
- $R, L, G, C$  are for total line length  $l$

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$



# Shorted Stub Equivalent Circuit

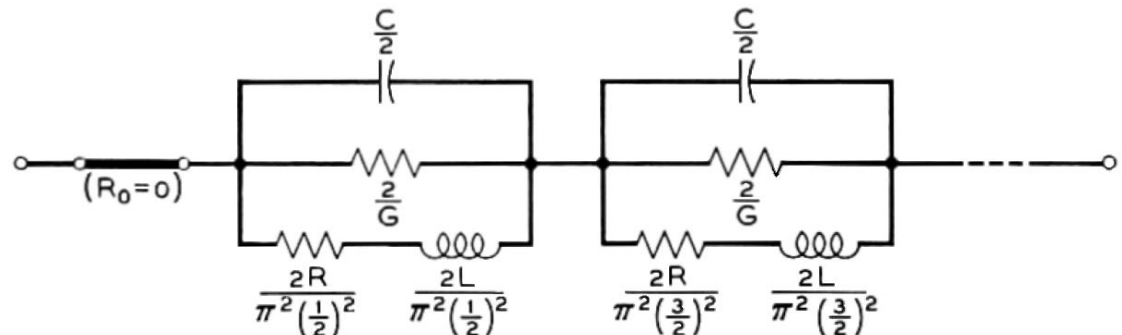
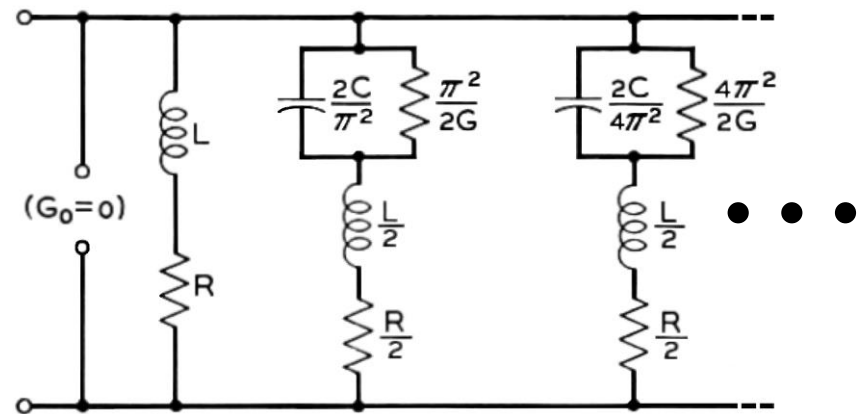


$$Z_{in} = Z_0 \tanh \gamma l = Z_0 \tanh (\alpha l + j\beta l)$$

- Loss is frequency-independent
- $R, L, G, C$  are for total line length  $l$

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

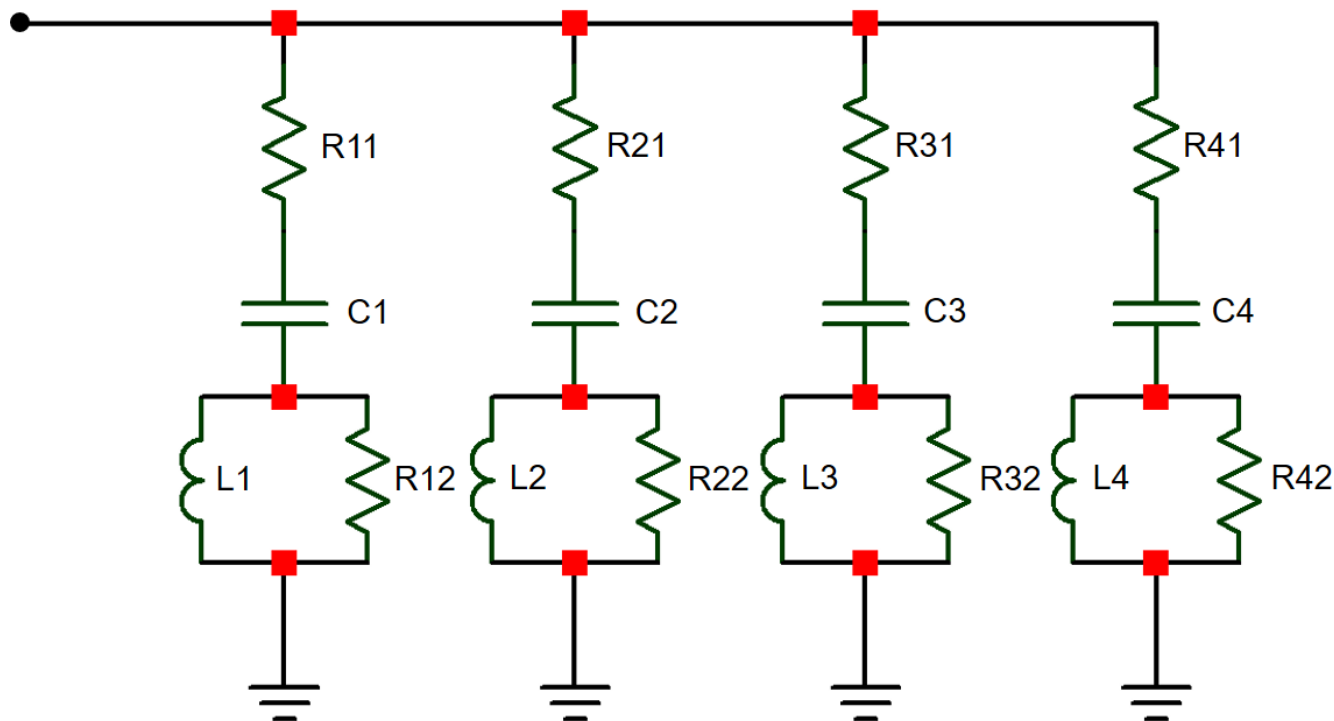
$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$





# 98.4-ft Dipole ( $L/d = 11,000$ )

## Equivalent Circuit for Zero to 30 MHz



$$R11 = 5.06 \, \Omega$$

$$C1 = 39.9 \, \text{pF}$$

$$L1 = 27.1 \, \mu\text{H}$$

$$R12 = 10.1 \, \text{k}\Omega$$

$$R21 = 0 \, \Omega$$

$$C2 = 4.64 \, \text{pF}$$

$$L2 = 24.9 \, \mu\text{H}$$

$$R22 = 50.1 \, \text{k}\Omega$$

$$R31 = 25.5 \, \Omega$$

$$C3 = 4.69 \, \text{pF}$$

$$L3 = 2.26 \, \mu\text{H}$$

$$R32 = 2.68 \, \text{k}\Omega$$

$$R41 = 0 \, \Omega$$

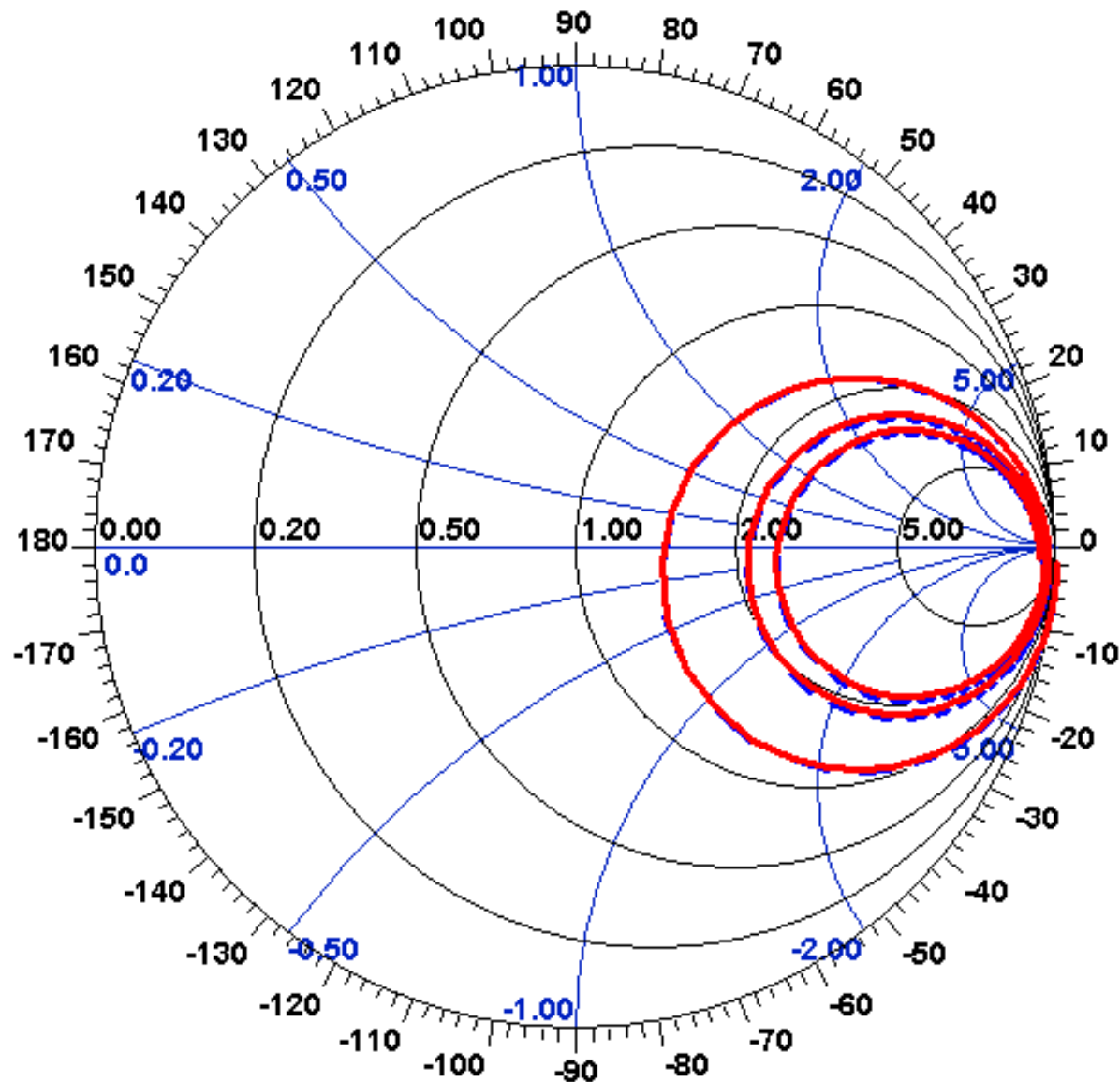
$$C4 = 1.68 \, \text{pF}$$

$$L4 = 24.5 \, \mu\text{H}$$

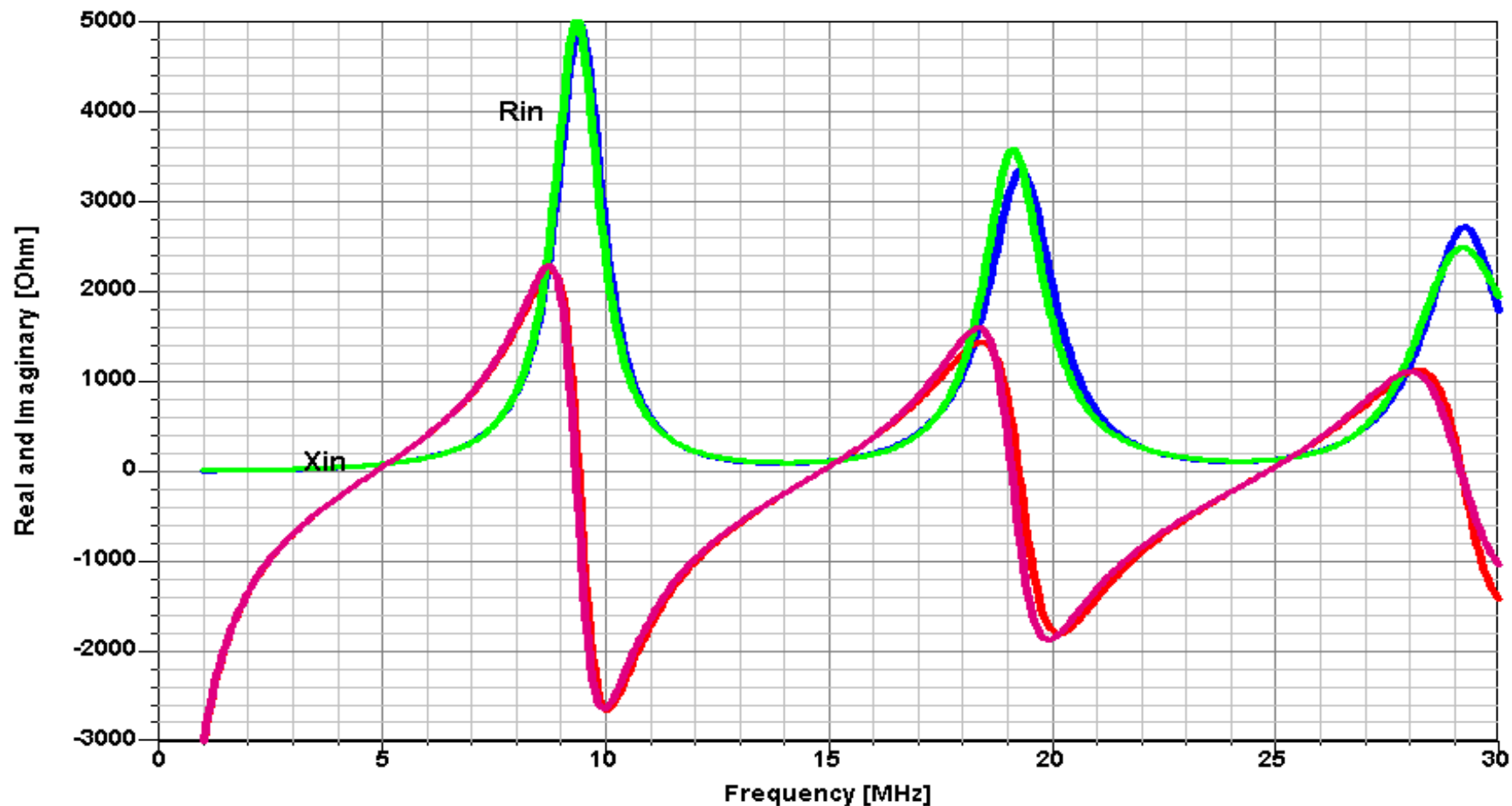
$$R42 = 116 \, \text{k}\Omega$$

# 98.4-ft Dipole

## Equivalent Circuit Performance from Zero to 30 MHz



# 98.4-ft Dipole Equivalent Circuit Impedance



# QucsStudio

QucsStudio 4.2.2 - Project: Dipole\_1m

File Edit Positioning Insert Project Tools Simulation View Help

Schematic: sch Schematic: dpl

**Optimization**

Opt1  
Sim=SP1  
Nelder-Mead|30000|1e-10|0.05|1  
C1=0...3.48817e-12...7e-12 linear  
C2=0...4.45802e-13...1e-12 linear  
C3=0...1.75184e-13...1e-12 linear  
C4=0...1.3793e-12...3e-12 linear  
C5=0...5.19777e-14...1e-13 linear  
C6=0...8.78836e-14...2e-13 linear  
C7=0...6.76868e-14...2e-13 linear  
L1=0...4.03083e-07...1e-06 linear  
L2=0...3.12801e-07...1e-06 linear  
L3=0...2.78706e-07...1e-06 linear  
L4=0...1.80955e-08...1e-07 linear  
L5=0...2.81925e-07...1e-06 linear  
L6=0...2.79429e-07...1e-06 linear  
L7=0...1.26848e-07...1e-06 linear  
R11=inactive  
R21=inactive  
R31=inactive  
R41=inactive  
R51=0...160.357...320 linear  
R61=inactive  
R71=0...0.828002...1 linear  
R12=0...1639.59...3200 linear  
R22=0...6427.51...13000 linear  
R32=0...12189.1...25000 linear  
R42=0...14.5389...25 linear  
R52=inactive  
R62=0...21846.8...40000 linear  
R72=0...14620.8...30000 linear  
Mean\_Square\_Error=1.89 MIN  
Max\_Square\_Error=1 MIN  
RMS\_Ang\_Error=0.000343 MIN

antdata\_S\_RI  
File=HOBBIES\_Ld50\_20segs\_S\_RI.sip  
Ports=1  
domain=rectangular  
interpol=cubic

**s-parameter simulation**

SP1  
Type=lin  
Start=1 MHz  
Stop=1500 MHz  
Points=1500

**equation**

Eqn1  
Rant=real((1+S[1,1])/(1-S[1,1]))\*50  
Xant=imag((1+S[1,1])/(1-S[1,1]))\*50  
Req=real((1+S[2,2])/(1-S[2,2]))\*50  
Xeq=imag((1+S[2,2])/(1-S[2,2]))\*50

**equation**

Goals  
Mean\_Square\_Error=average(range((mag(S[1,1]-S[2,2]))^2, 1MHz, 1500MHz))  
Max\_Square\_Error=max(range((mag(S[1,1]-S[2,2]))^2, 1MHz, 1500MHz))  
RMS\_Ang\_Error=sqrt(average(range((mag(phase(S[1,1]-S[2,2]))^2, 1MHz, 1500MHz)))

| Mean_Square_Error | Max_Square_Error | RMS_Ang_Error |
|-------------------|------------------|---------------|
| 0.000127          | 0.000238         | 0.7           |

Combined Goal Value

Optimizer Iteration Number

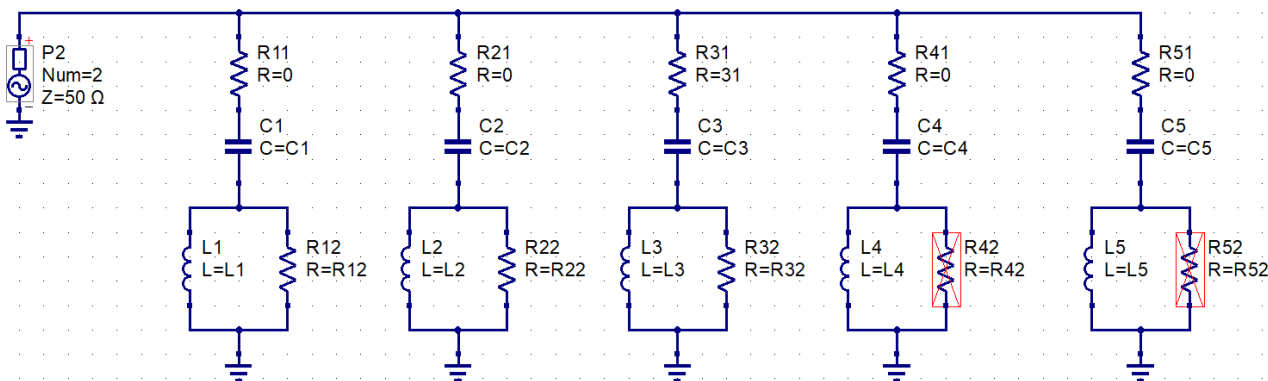
no warnings 0:0

# Importing Data into QucsStudio

- Antenna modeling programs compute impedance  $Z$  data directly in real and imaginary form as  $R$  and  $X$
- $Z$  data should be converted to  $S$  data in a Touchstone .s1p file
- QucsStudio accepts Touchstone files in MA, dB, and RI forms
- RI and dB are more accurate than MA if points are near the boundary of the Smith chart
- Import data files into a project one of two ways
  - Use the “Project → Add File to Project” command, or
  - Copy/paste the file to a project folder before opening the project
- Under the **Components** tab, choose the **Devices** category; place an  $N$ -port on the schematic; open it and set the file name and number of ports

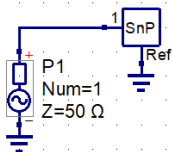
Best results are achieved by importing and interpolating  $S$  data in rectangular coordinates with a cubic interpolator.

# VHF-UHF Discone Broadband Equivalent Circuit – Optimization Setup



antdata\_S\_MA  
File=VHF-UHF\_Discone\_S\_MA.s1p  
Ports=1  
domain=rectangular  
interpol=cubic

| F1 MHz | F2 MHz | F3 MHz | F4 MHz   | F5 MHz   |
|--------|--------|--------|----------|----------|
| 288    | 471    | 666    | 1.37e+03 | 1.81e+06 |



s-parameter  
simulation

SP1  
Type=lin  
Start=100 MHz  
Stop=1000 MHz  
Points=901

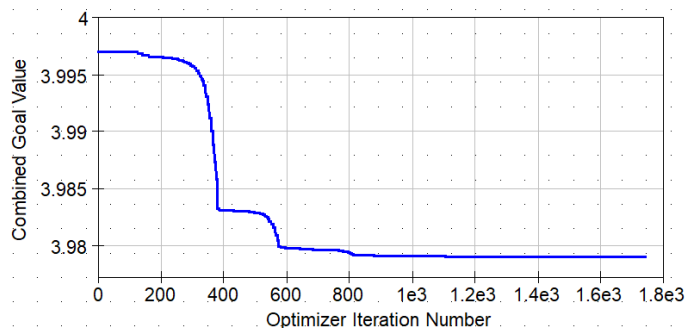
equation

Outputs  
 $Rant = \text{real}((1+S[1,1])/(1-S[1,1]))*50$   
 $Xant = \text{imag}((1+S[1,1])/(1-S[1,1]))*50$   
 $Gant = \text{real}((1-S[1,1])/(1+S[1,1]))*50$   
 $Bant = \text{imag}((1-S[1,1])/(1+S[1,1]))*50$   
 $Req = \text{real}((1+S[2,2])/(1-S[2,2]))*50$   
 $Xeq = \text{imag}((1+S[2,2])/(1-S[2,2]))*50$   
 $Geq = \text{real}((1-S[2,2])/(1+S[2,2]))*50$   
 $Beq = \text{imag}((1-S[2,2])/(1+S[2,2]))*50$   
 $F1 = 1e-6/(2*\pi*\text{sqrt}(L1*C1))$   
 $F2 = 1e-6/(2*\pi*\text{sqrt}(L2*C2))$   
 $F3 = 1e-6/(2*\pi*\text{sqrt}(L3*C3))$   
 $F4 = 1e-6/(2*\pi*\text{sqrt}(L4*C4))$   
 $F5 = 1e-6/(2*\pi*\text{sqrt}(L5*C5))$

equation

Goals  
 $\text{Mean\_Square\_Error\_S\_Mag} = \text{average}(\text{range}((\text{mag}(S[1,1]-S[2,2]))^2, 100\text{MHz}, 1000\text{MHz}))$   
 $\text{Mean\_Square\_Error\_S\_Ang} = \text{average}(\text{range}((\text{mag}(\text{phase}(S[1,1]-S[2,2])))^2, 100\text{MHz}, 1000\text{MHz}))$   
 $\text{Max\_Square\_Error\_S\_Mag} = \text{max}(\text{range}((\text{mag}(S[1,1]-S[2,2]))^2, 100\text{MHz}, 1000\text{MHz}))$   
 $\text{Max\_Square\_Error\_S\_Ang} = \text{max}(\text{range}((\text{mag}(\text{phase}(S[1,1]-S[2,2])))^2, 100\text{MHz}, 1000\text{MHz}))$

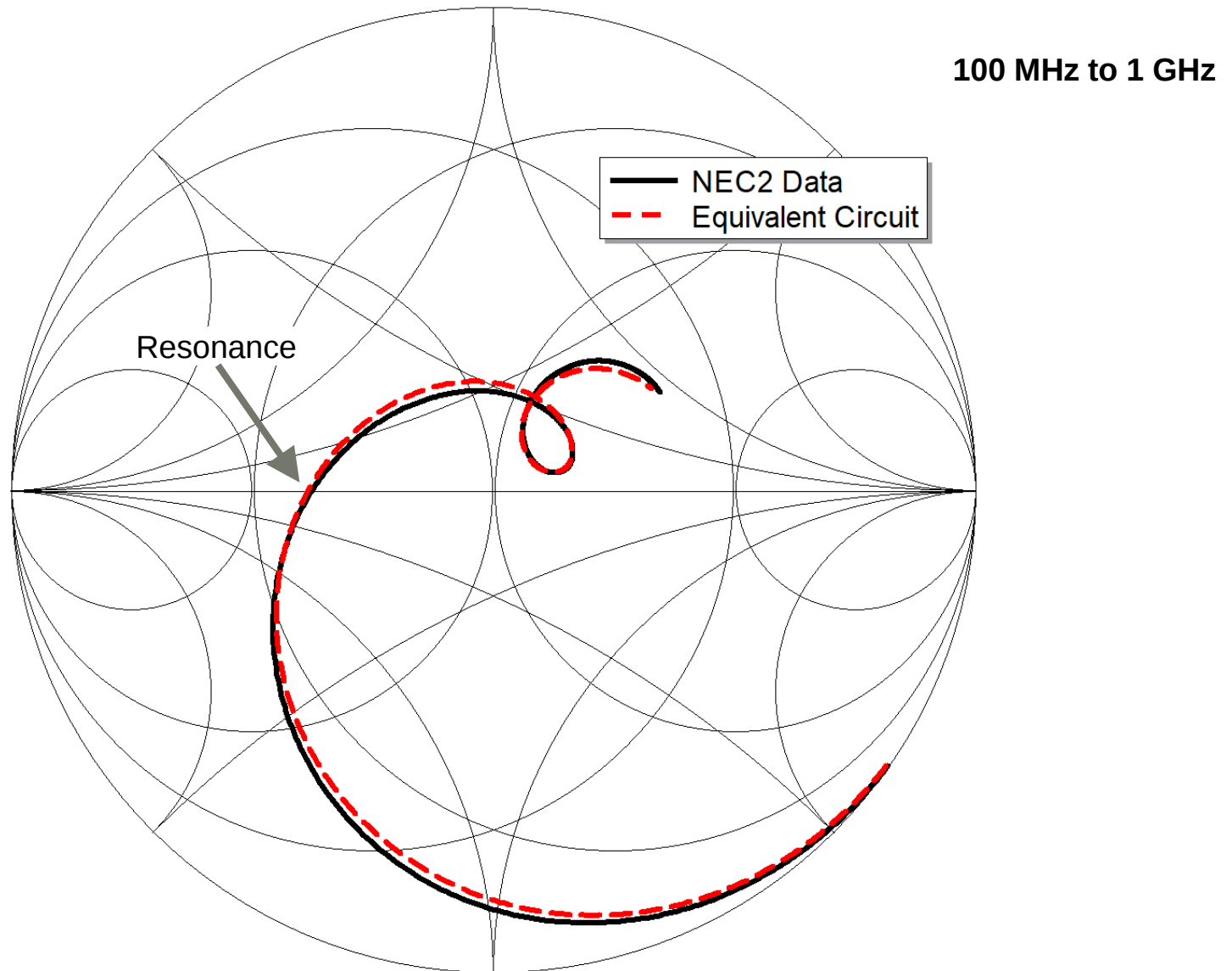
| MeanSE  S | MeanSE Ang(S) | MaxSE  S | MaxSE Ang(S) |
|-----------|---------------|----------|--------------|
| 0.000196  | 2.51          | 0.000411 | 6.52         |



Optimization

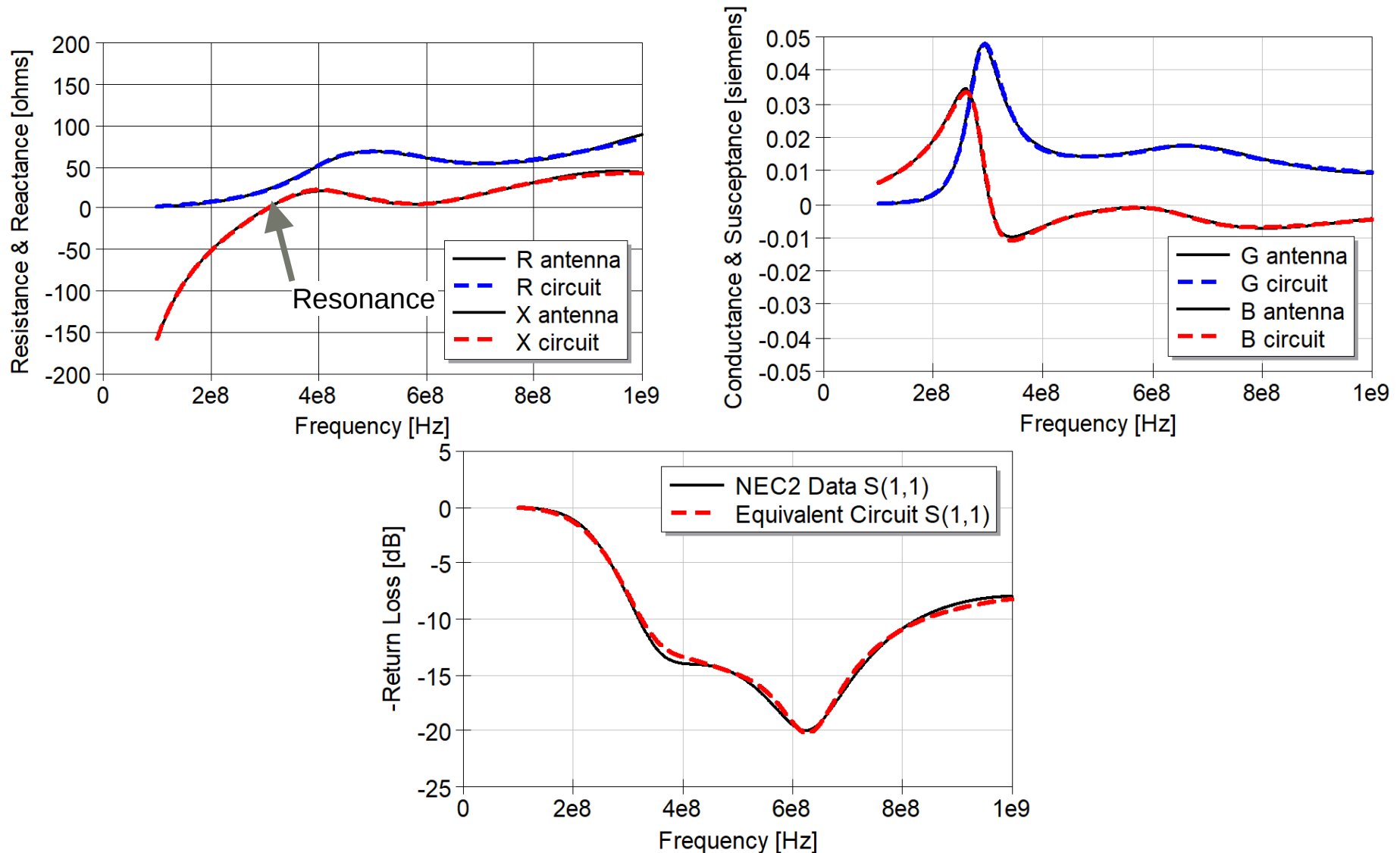
Opt1  
 Sim=SP1  
 Nelder-Mead|40000|1e-12|[0.01]1  
 $C1 = 0 \dots 7.21944e-12 \dots 7.3e-11$  linear  
 $C2 = 0 \dots 7.34134e-13 \dots 7.4e-12$  linear  
 $C3 = 0 \dots 9.07243e-13 \dots 9.2e-12$  linear  
 $C4 = 0 \dots 1.7945e-13 \dots 1.9e-12$  linear  
 $C5 = 0 \dots 3.56588e-17 \dots 2.3e-12$  linear  
 $L1 = 0 \dots 4.22938e-08 \dots 4.2e-07$  linear  
 $L2 = 0 \dots 1.5529e-07 \dots 1.5e-06$  linear  
 $L3 = 0 \dots 6.29127e-08 \dots 6.1e-07$  linear  
 $L4 = 0 \dots 7.5276e-08 \dots 7.3e-07$  linear  
 $L5 = 0 \dots 2.17049e-10 \dots 1.2e-08$  linear  
 $R11 = \text{inactive}$   
 $R21 = \text{inactive}$   
 $R31 = 0 \dots 97.7606 \dots 320$  linear  
 $R41 = \text{inactive}$   
 $R51 = \text{inactive}$   
 $R12 = 0 \dots 271.895 \dots 2700$  linear  
 $R22 = 0 \dots 824.449 \dots 8100$  linear  
 $R32 = 0 \dots 842.584 \dots 7900$  linear  
 $R42 = \text{inactive}$   
 $R52 = \text{inactive}$   
 $\text{Mean\_Square\_Error\_S\_Mag} = 5115$  MIN  
 $\text{Mean\_Square\_Error\_S\_Ang} = 0.395$  MIN  
 $\text{Max\_Square\_Error\_S\_Mag} = 2434$  MIN  
 $\text{Max\_Square\_Error\_S\_Ang} = 0.151$  MIN

# Discone Equivalent Circuit Performance

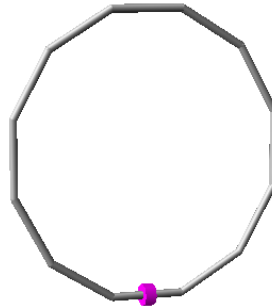




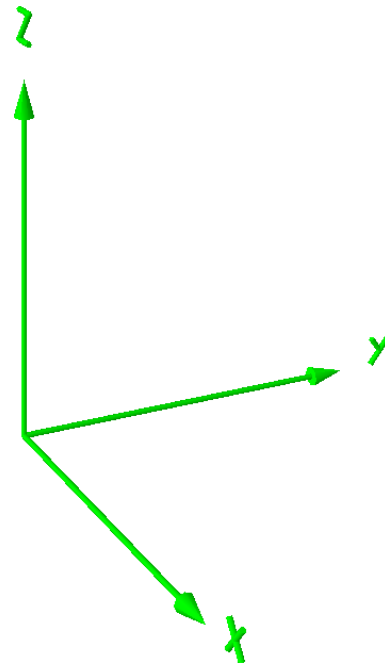
# Discone Equivalent Circuit Performance



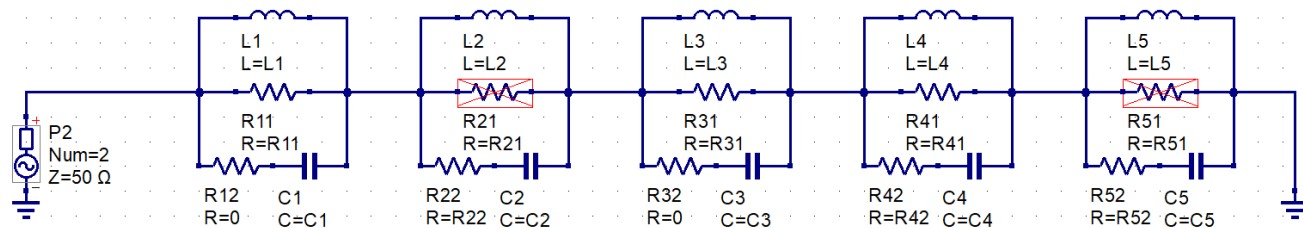
# 4nec2 20-Meter Single-Turn Circular Loop



- Model: LoopCirc20
- Loop diameter: 23.4 ft
- Wire diameter: AWG #12
- Free space, no ground



# 20-Meter Single-Turn Circular Loop Type 1 Equivalent Circuit – Optimization Setup



## Optimization

Opt1  
Sim=SP1  
Nelder-Mead[40000][1e-12][0.05][1  
C1=0...2.21109e-10...2.0e-09 linear  
C2=0...2.13425e-11...2.2e-10 linear  
C3=0...2.18256e-11...2.2e-10 linear  
C4=0...1.06428e-11...1.1e-10 linear  
C5=0...5.91074e-16...3.3e-12 linear  
L1=0...3.19847e-06...3.0e-05 linear  
L2=0...2.80178e-05...2.8e-04 linear  
L3=0...2.89529e-06...2.9e-05 linear  
L4=0...1.83867e-06...1.9e-05 linear  
L5=0...3.46573e-07...3.1e-06 linear  
R11=0...116.29...1e5 linear  
R21=inactive  
R31=0...4774.67...1e5 linear  
R41=0...10889.7...1e5 linear  
R51=inactive  
R12=inactive  
R22=0...32.3559...100 linear  
R32=inactive  
R42=0...8.30104...50 linear  
R52=0...30.5479...1000 linear  
Mean\_Square\_Error\_S\_Mag=20750 MIN  
Mean\_Square\_Error\_S\_Ang=13.35 MIN  
Max\_Square\_Error\_S\_Mag=3534 MIN  
Max\_Square\_Error\_S\_Ang=0.806 MIN

antdata\_S\_MA  
File=LoopCirc20\_S11\_dB.s1p  
Ports=1  
domain=rectangular  
interpol=cubic

| F1 MHz | F2 MHz | F3 MHz | F4 MHz | F5 MHz |
|--------|--------|--------|--------|--------|
| 5.842  | 6.516  | 20.01  | 35.98  |        |

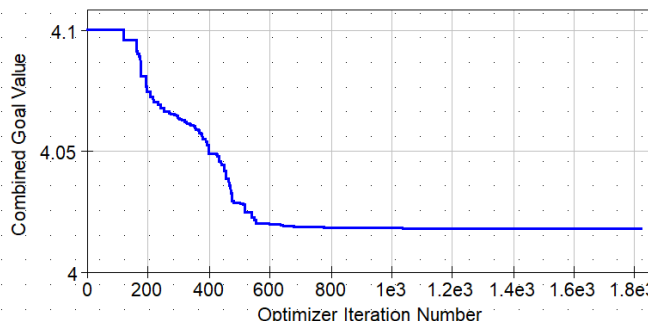
## equation

Outputs  
Rant=real((1+S[1,1])/(1-S[1,1]))\*50  
Xant=imag((1+S[1,1])/(1-S[1,1]))\*50  
Gant=real((1-S[1,1])/(1+S[1,1]))\*50  
Bant=imag((1-S[1,1])/(1+S[1,1]))\*50  
Req=real((1+S[2,2])/(1-S[2,2]))\*50  
Xeq=imag((1+S[2,2])/(1-S[2,2]))\*50  
Geq=real((1-S[2,2])/(1+S[2,2]))\*50  
Beq=imag((1-S[2,2])/(1+S[2,2]))\*50  
F1=1e-06/(2\*pi\*sqrt(L1\*C1))  
F2=1e-06/(2\*pi\*sqrt(L2\*C2))  
F3=1e-06/(2\*pi\*sqrt(L3\*C3))  
F4=1e-06/(2\*pi\*sqrt(L4\*C4))  
F5=1e-06/(2\*pi\*sqrt(L5\*C5))

## equation

Goals  
Mean\_Square\_Error\_S\_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Mean\_Square\_Error\_S\_Ang=average(range((mag(phase(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Max\_Square\_Error\_S\_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Max\_Square\_Error\_S\_Ang=max(range((mag(phase(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))

| MeanSE  S | MeanSE Ang(S) | MaxSE  S | MaxSE Ang(S) |
|-----------|---------------|----------|--------------|
| 4.83e-05  | 0.0796        | 0.00028  | 1.19         |

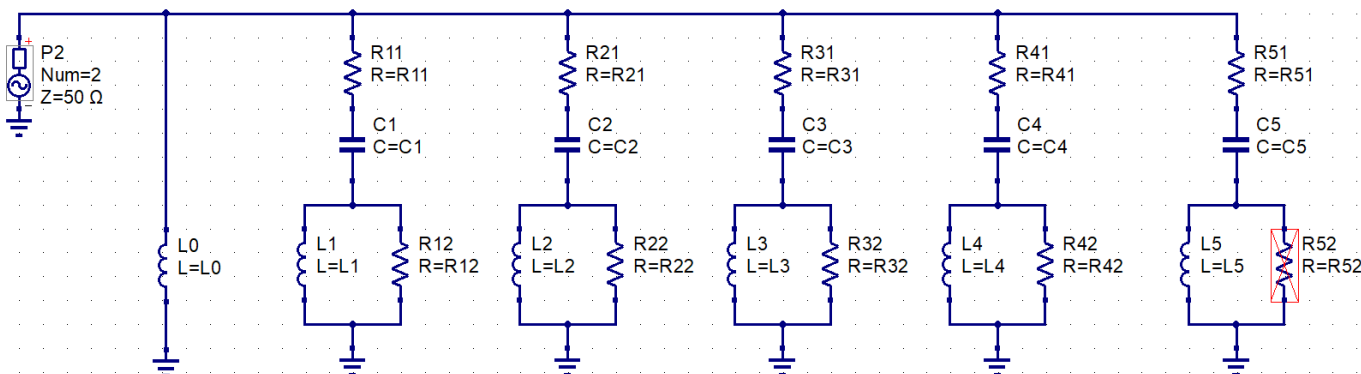


## s-parameter simulation

SP1  
Type=log  
Start=0.01 MHz  
Stop=30 MHz  
Points=3000

# 20-Meter Single-Turn Circular Loop

## Type 2 Equivalent Circuit – Optimization Setup



### Optimization

Opt1  
Sim=SP1  
Nelder-Mead|40000|1e-12|0.01|1  
C1=0...7.86414e-12...7.9e-11 linear  
C2=0...1.59942e-12...1.6e-11 linear  
C3=0...4.91637e-13...4.9e-12 linear  
C4=0...2.03925e-12...2.0e-11 linear  
C5=0...1.0146e-12...1.0e-11 linear  
L0=0...3.62962e-05...3.6e-04 linear  
L1=0...1.64092e-05...1.6e-04 linear  
L2=0...2.06872e-05...2.1e-04 linear  
L3=0...6.56726e-05...6.6e-04 linear  
L4=0...6.48313e-06...6.5e-05 linear  
L5=0...2.56029e-08...2.5e-07 linear  
R11=0...56.8399...570 linear  
R21=0...123.956...1200 linear  
R31=0...1.55294...16 linear  
R41=0...0.132956...1.3 linear  
R51=0...4.6259...41 linear  
R12=0...26336.7...2.6e05 linear  
R22=0...130900...1.3e06 linear  
R32=0...157049...1.5e06 linear  
R42=0...88003.2...1.0e6 linear  
R52=inactive  
Mean\_Square\_Error\_S\_Mag=474000 MIN  
Mean\_Square\_Error\_S\_Ang=559 MIN  
Max\_Square\_Error\_S\_Mag=108000 MIN  
Max\_Square\_Error\_S\_Ang=125 MIN

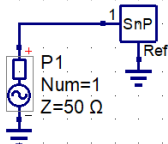
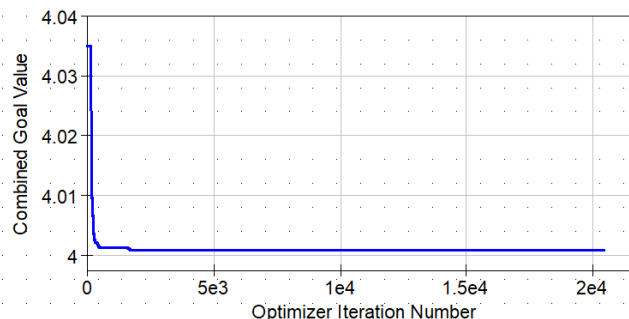
antdata\_S\_MA  
File=LoopCirc20\_S11\_dB.s1p  
Ports=1  
domain=rectangular  
interpol=cubic

| F1 MHz | F2 MHz | F3 MHz | F4 MHz | F5 MHz |
|--------|--------|--------|--------|--------|
| 14.01  | 27.67  | 28.01  | 43.77  | 987.5  |

### equation

Goals  
Mean\_Square\_Error\_S\_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Mean\_Square\_Error\_S\_Ang=average(range((mag(phase(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Max\_Square\_Error\_S\_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))  
Max\_Square\_Error\_S\_Ang=max(range((mag(phase(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))

| MeanSE  S | MeanSE Ang(S) | MaxSE  S | MaxSE Ang(S) |
|-----------|---------------|----------|--------------|
| 2.11e-06  | 0.00179       | 9.31e-06 | 0.00797      |



### s-parameter simulation

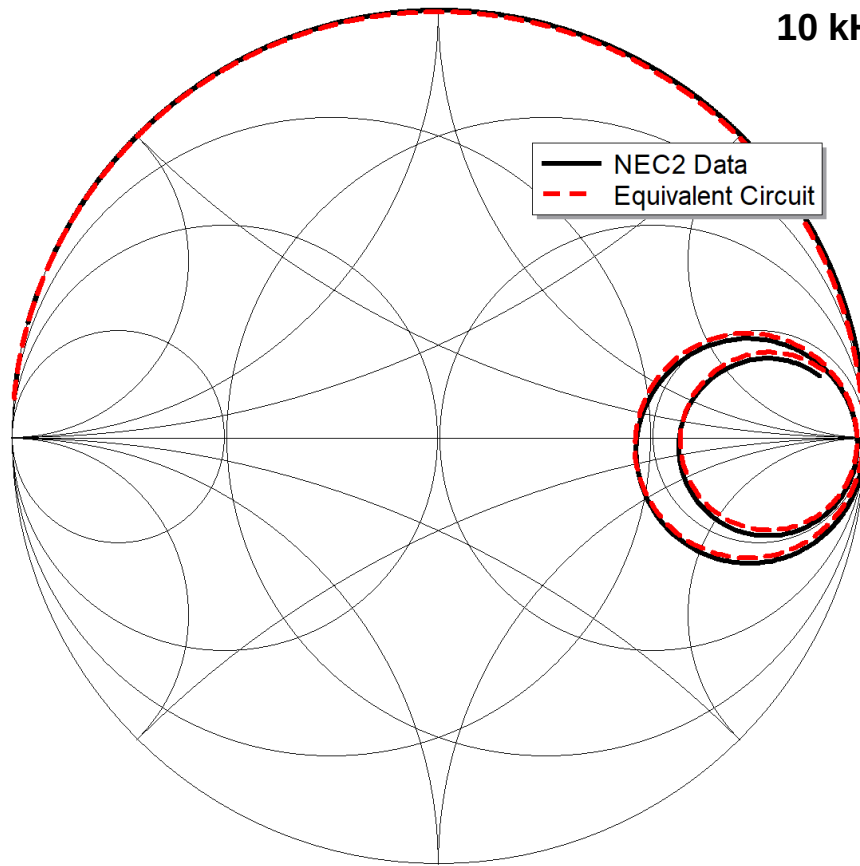
SP1  
Type=lin  
Start=0.01 MHz  
Stop=30 MHz  
Points=3000

### equation

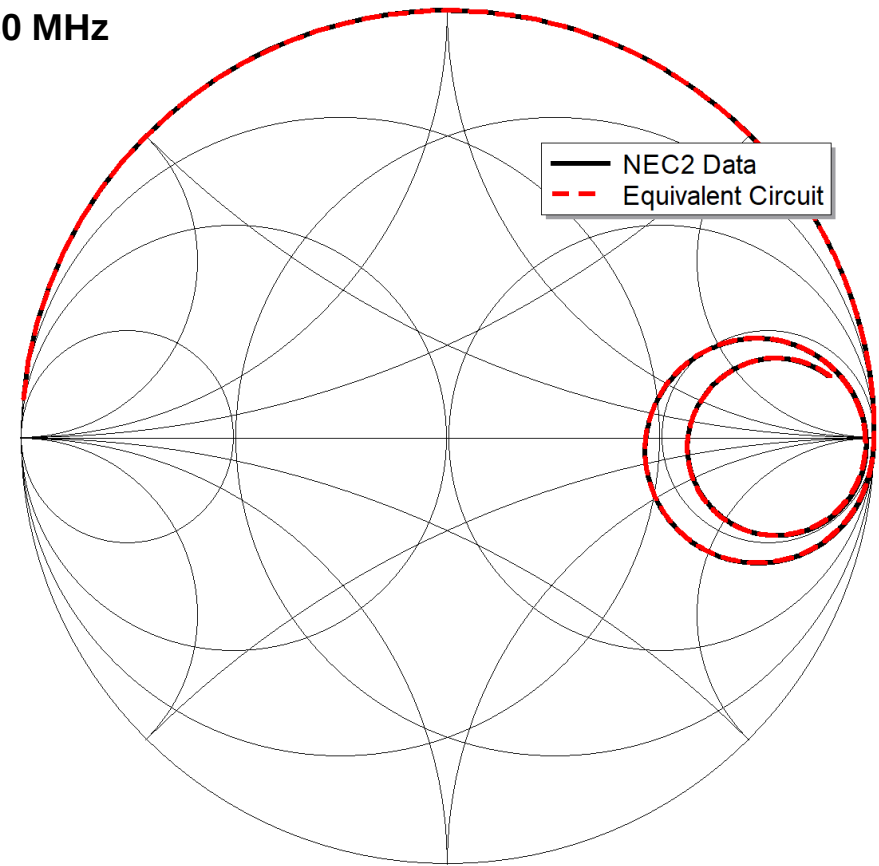
Outputs  
Rant=real((1+S[1,1])/(1-S[1,1]))\*50  
Xant=imag((1+S[1,1])/(1-S[1,1]))\*50  
Gant=real((1-S[1,1])/(1+S[1,1]))/50  
Bant=imag((1-S[1,1])/(1+S[1,1]))/50  
Req=real((1+S[2,2])/(1-S[2,2]))\*50  
Xeq=imag((1+S[2,2])/(1-S[2,2]))\*50  
Geq=real((1-S[2,2])/(1+S[2,2]))/50  
Beq=imag((1-S[2,2])/(1+S[2,2]))/50  
F1=1e-06/(2\*pi\*sqrt(L1\*C1))  
F2=1e-06/(2\*pi\*sqrt(L2\*C2))  
F3=1e-06/(2\*pi\*sqrt(L3\*C3))  
F4=1e-06/(2\*pi\*sqrt(L4\*C4))  
F5=1e-06/(2\*pi\*sqrt(L5\*C5))

# 20-Meter Circular Loop Equivalent Circuit Performance

## Type 1 Series Ladder

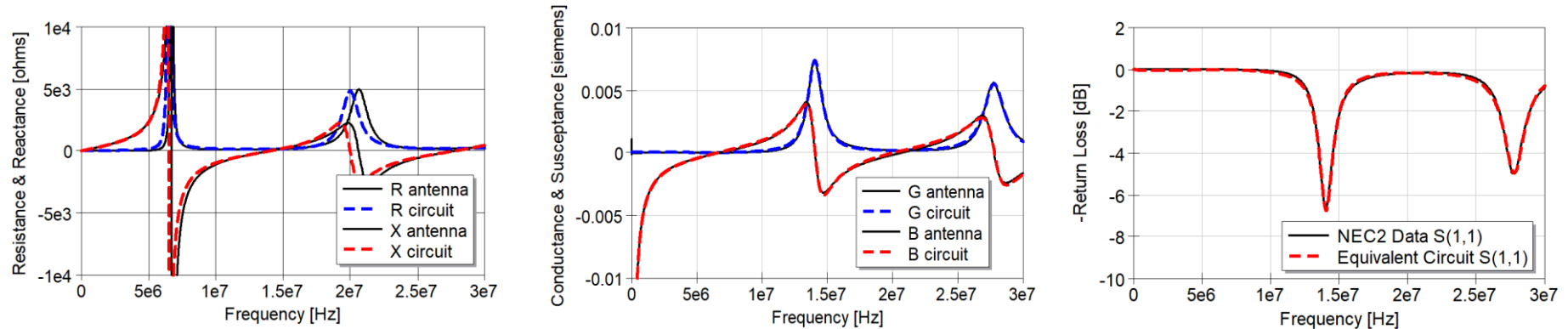


## Type 2 Parallel Ladder

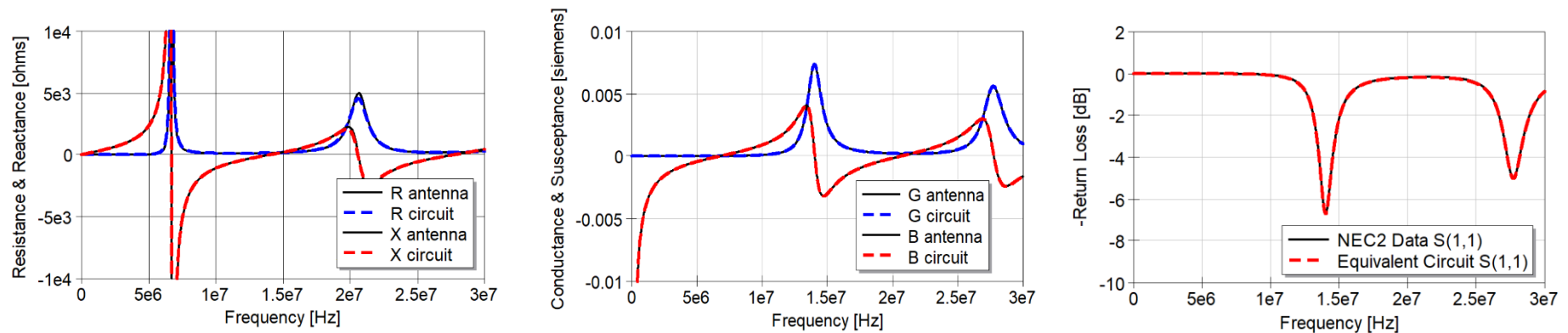


# 20-Meter Circular Loop Equivalent Circuit Performance

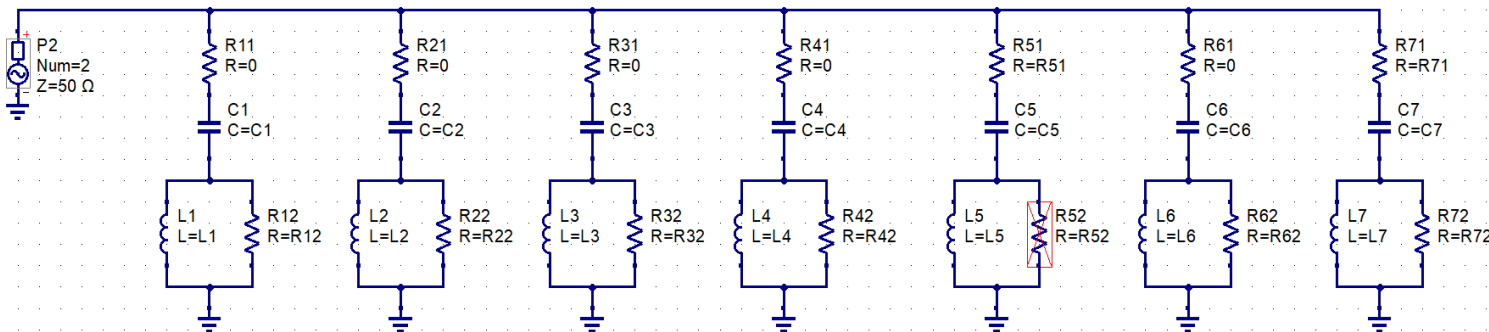
## ■ Type 1 Equivalent Circuit



## ■ Type 2 Equivalent Circuit



# 1-Meter Long Dipole Broadband Equivalent Circuit – Optimization Setup



antdata\_S\_RI  
File=HOBBIES\_Ld50\_20segs\_S\_RI.s1p  
Ports=1  
domain=rectangular  
interpol=cubic

| F1 MHz   | F2 MHz   | F3 MHz  | F4 MHz   | F5 MHz   | F6 MHz   | F7 MHz   |
|----------|----------|---------|----------|----------|----------|----------|
| 1.34e+08 | 4.26e+08 | 7.2e+08 | 9.93e+08 | 1.31e+09 | 1.02e+09 | 1.71e+09 |

## equation

### Outputs

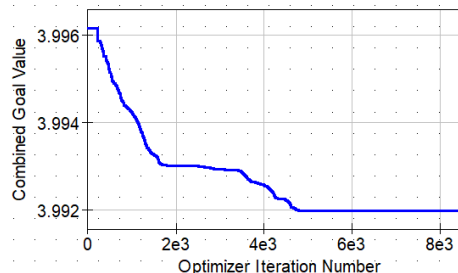
Rant=real((1+S[1,1])/(1-S[1,1]))\*50  
Xant=imag((1+S[1,1])/(1-S[1,1]))\*50  
Gant=real((1-S[1,1])/(1+S[1,1]))/50  
Bant=imag((1-S[1,1])/(1+S[1,1]))/50  
Req=real((1+S[2,2])/(1-S[2,2]))\*50  
Xeq=imag((1+S[2,2])/(1-S[2,2]))\*50  
Geq=real((1-S[2,2])/(1+S[2,2]))/50  
Beq=imag((1-S[2,2])/(1+S[2,2]))/50  
F1=1/(2\*pi\*sqrt(C1\*L1))  
F2=1/(2\*pi\*sqrt(C2\*L2))  
F3=1/(2\*pi\*sqrt(C3\*L3))  
F4=1/(2\*pi\*sqrt(C4\*L4))  
F5=1/(2\*pi\*sqrt(C5\*L5))  
F6=1/(2\*pi\*sqrt(C6\*L6))  
F7=1/(2\*pi\*sqrt(C7\*L7))

## equation

### Goals

Mean\_Square\_Error\_S\_Mag=average(range((mag(S[1,1]/S[2,2]))^2, 1MHz, 1500MHz))  
Mean\_Square\_Error\_S\_Ang=average(range((mag(phase(S[1,1]/S[2,2]))^2, 1MHz, 1500MHz))  
Max\_Square\_Error\_S\_Mag=max(range((mag(S[1,1]/S[2,2]))^2, 1MHz, 1500MHz))  
Max\_Square\_Error\_S\_Ang=max(range((mag(phase(S[1,1]/S[2,2]))^2, 1MHz, 1500MHz))

| MeanSE  S | MeanSE Ang(S) | MaxSE  S | MaxSE Ang(S) |
|-----------|---------------|----------|--------------|
| 0.000132  | 0.48          | 0.000253 | 2.1          |



## Optimization

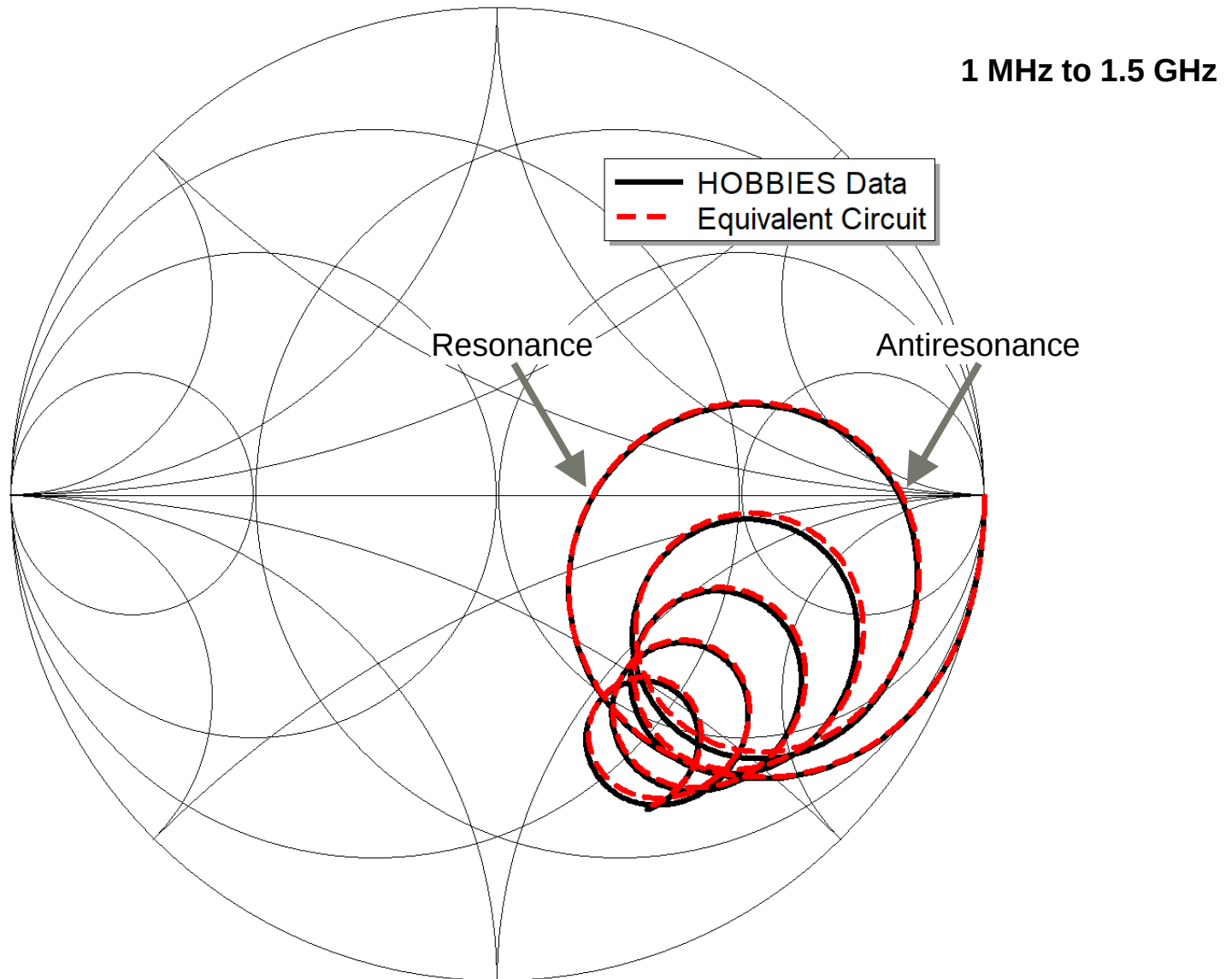
Opt1  
Sim=SP1  
Nelder-Mead[40000][1e-12][0.01]1  
C1=0...3.47959e-12...3.5e-11 linear  
C2=0...4.45923e-13...4.5e-12 linear  
C3=0...1.75036e-13...1.8e-12 linear  
C4=0...1.38164e-12...1.3e-11 linear  
C5=0...5.19929e-14...5.2e-13 linear  
C6=0...8.80583e-14...8.8e-13 linear  
C7=0...6.70298e-14...6.7e-13 linear  
L1=0...4.04026e-07...4.0e-06 linear  
L2=0...3.13163e-07...3.1e-06 linear  
L3=0...2.79047e-07...2.8e-06 linear  
L4=0...1.85856e-08...1.8e-07 linear  
L5=0...2.81936e-07...2.8e-06 linear  
L6=0...2.78965e-07...2.8e-06 linear  
L7=0...1.28491e-07...1.3e-06 linear  
R11=inactive  
R21=inactive  
R31=inactive  
R41=inactive  
R51=0...160.862...320 linear  
R61=inactive  
R71=0...0.973613...10 linear  
R12=0...1649.96...3200 linear  
R22=0...6428.04...13000 linear  
R32=0...12227.8...25000 linear  
R42=0...14.4632...30 linear  
R52=inactive  
R62=0...21736.1...45000 linear  
R72=0...14402.2...30000 linear  
Mean\_Square\_Error\_S\_Mag=7565 MIN  
Mean\_Square\_Error\_S\_Ang=2.06 MIN  
Max\_Square\_Error\_S\_Mag=3955 MIN  
Max\_Square\_Error\_S\_Ang=0.477 MIN

## s-parameter simulation

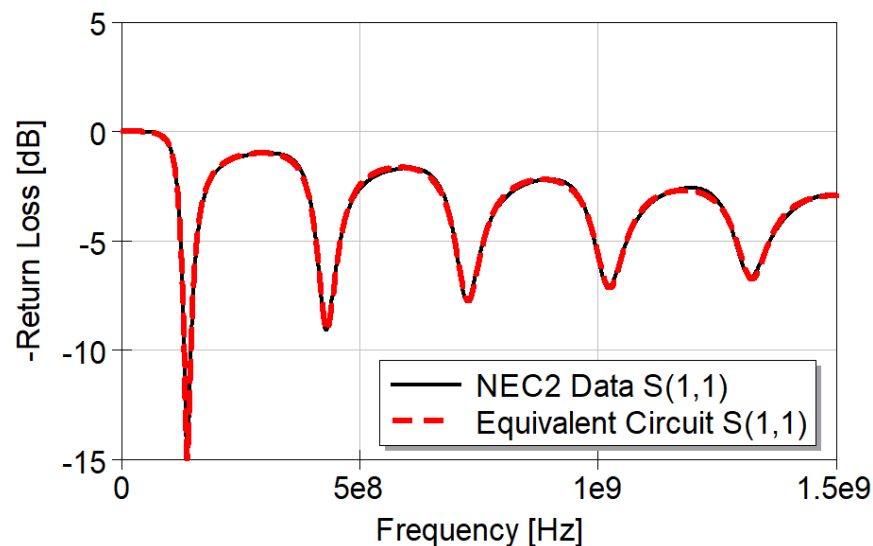
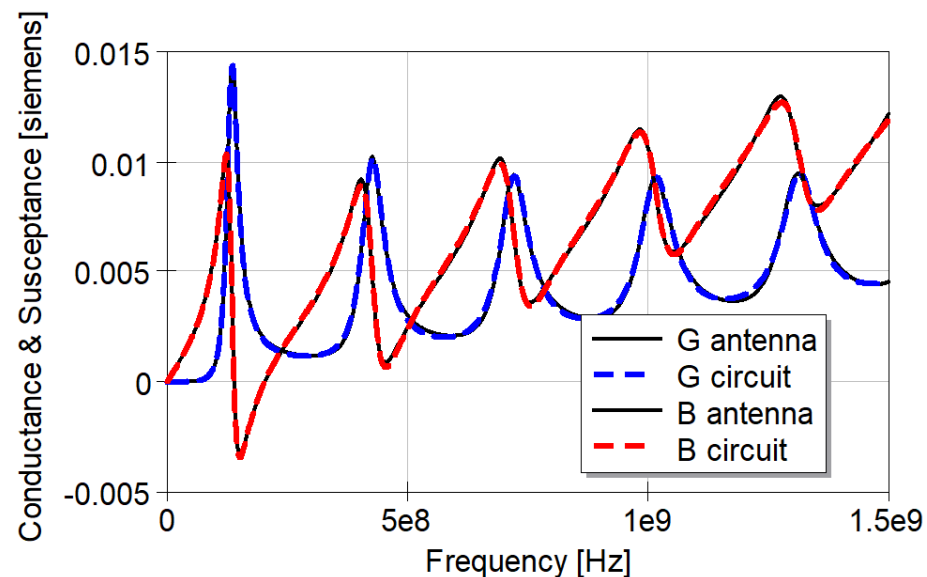
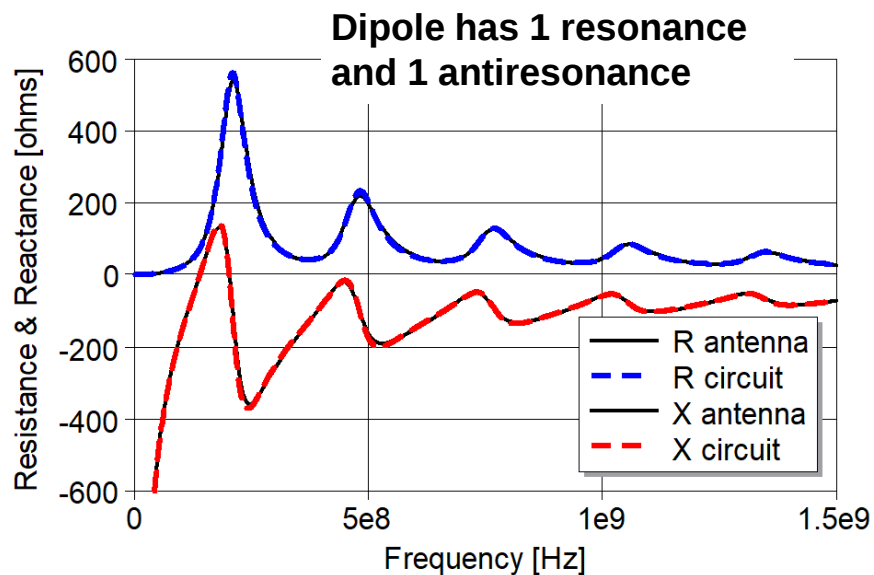
SP1  
Type=lin  
Start=1 MHz  
Stop=1500 MHz  
Points=1500



# 1-Meter Dipole Equivalent Circuit Performance



# 1-Meter Dipole Equivalent Circuit Performance



# Summary and Conclusions

- **Immittance (admittance or impedance) of 1-port distributed electromagnetic structures like antennas have broadband lumped-element equivalent circuits**
- **Antenna impedance can always be represented by a generalized ladder network of just four possible topologies**
  - Number of stages depends on desired accuracy of approximation
- **Element values may be set by either of two methods**
  - Semi-analytical: Compute element values from the poles and residues found by SEM analysis
  - Numerical: Use a circuit optimizer to fit one of four ladder circuits to measured or computed impedance data. Fit a small bandwidth first. Add stages to increase bandwidth
- **If making a 2-port antenna emulator for non-radiating transmission testing**
  - Use Darlington synthesis to convert the network to a reactance 2-port and Port 2 termination resistor
  - Load resistor at Port 2 represents radiation plus loss

---

# **Resources**

## **Software & Books**

# FARS Radio Links Page

<https://www.fars.k6ya.org/others>

QucsStudio



## Software

de **K6OIK**

### General Circuit Analysis and Design

- ▶ [Micro-Cap](#) by Spectrum Software [free download].
- ▶ [RF resources](#) by Gunthard Kraus DG8GB. Choose English. A nice selection of articles, projects, and downloadable software [free].
- ▶ [QucsStudio](#) Circuit Simulator, [free download]
- ▶ [OrCAD](#), demo version, [free trial]
- ▶ [LTspice](#) Linear Technology, [free download]

### Filters (lumped element ladders)

- ▶ [Elsie](#), Jim Tonne, WB6BLD, free student edition

### Inductors

- ▶ [NBS](#) coil design, Stephen Moshier, [free download]

### Transmission Lines

- ▶ [Times Microwave online coax calculator](#) [free download]
- ▶ [TX-Line](#), Applied Wave Research, [free download]
- ▶ [TLDetails](#), Dan Maguire, AC6LA, [free download]

### Smith Charts, Match Networks, and Tuners

- ▶ [SimSmith](#), Ward Harriman, AE6TY, [free download]
- ▶ [Smith](#), Fritz Dellsperger, HB9AJY, [free download]
- ▶ [QuickSmith](#), Nathan Iyer, KJ6FO, [free download]
- ▶ [JSmith](#), James Bromley, K7JEB, and James Tonne, W4ENE, [free download]
- ▶ [AMPSA MultiMatch](#) Impedance-Matching Wizard, Pieter Abrie, free trial
- ▶ [Nuhertz ZMatch](#), free trial
- ▶ [Optenni Lab](#), free trial
- ▶ TLW, Dean Straw, N6BV, On ARRL Antenna Book CD
- ▶ MATCH, William Sabin, W0IYH, On ARRL Handbook CD (chapter 17)
- ▶ [Impedance Matching Network Designer](#) (alternate link), John Wetherell, online calculator
- ▶ [T-Network Tuner](#) (alternate link), Kevin Schmidt, W9CF, [free download]

### Miscellaneous Routines

- ▶ [AppCAD](#), (alternate link) by Avago, [free download]

### Antennas

- ▶  **EM-Bench** [ANSim](#) is a moment method electromagnetic modeling program used mainly for antenna design. The thin-wire code can model electrically small and medium size structures made of wires, plates, and dielectric blocks. It is the main d
- ▶ [openEMS](#) Finite Difference Time Domain (FDTD) modeling code that uses Matlab or Octave, [free download]
- ▶ [4nec2](#), Antenna modeler by Arie Voors
- ▶ [EZNEC](#), Roy Lewallen, W7EL, \$89 (demo version with limited functionality, free)
- ▶ [Unofficial NEC Archives](#), Miscellaneous free NEC programs
- ▶ [FEKO Student Edition](#), part of HyperWorks 14 Student Edition [free download]
- ▶ [AutoEZ](#), Dan Maguire, AC6LA \$79 (demo version with limited functionality, free)
- ▶ [WIPL-D Demo](#) full function s/w with 30-day time limit ([free download])
- ▶ [WIPL-D Lite](#) limited function s/w for a smaller price
- ▶ [HOBBIES](#) Academic version - CD and License are included with the following book, which is the hard copy instruction manual: T.K. Sarkar, Higher Order Basis Based Integral Equation Solver (HOBBIES), Wiley, 2012, ISBN 1118140656. Price ranges from \$162 to \$210 depending on where you buy the book. HOBBIES is similar to WIPL-D but costs half as much, can handle bigger problems, and has a better user interface.

ANSim



HOBBIES



Coming Soon

AWAS

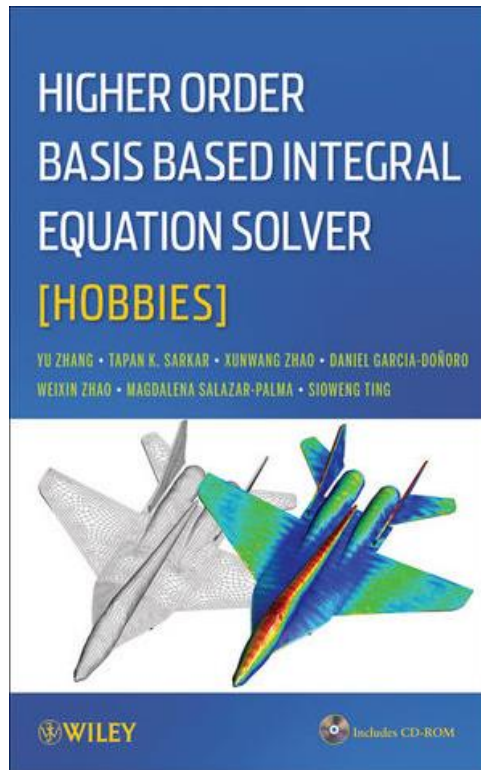
NEC5

# Antenna Modeling Programs for Radio Amateurs

- **ANSim 1.0.3** <https://www.em-bench.com/collections/all>
  - MBC method has better accuracy than NEC2 or NEC4 for wire antennas. Free basic version.
- **AWAS 2.0: contact K6OIK**
  - Popović higher-order polynomial thin-wire method found in WIPL-D and HOBBIES. Accurate Sommerfeld ground. Free.
- **HOBBIES** <https://www.hobbies-em.com> (New URL)
  - Similar to WIPL-D. Professional GUI based on GiD. Book includes Academic version. Book MSRP is \$240 but online prices vary widely.
- **NEC4 and NEC5** <https://ipo.llnl.gov/technologies/software/nec-v50-numerical-electromagnetic-code>
  - NEC5 Individual noncommercial user license \$110
  - NEC4 Individual noncommercial user license \$300
  - Free to students. Part of HyperWorks 2018 Student Edition
- **EZNEC** <https://www.eznec.com>
  - EZNEC v.6 Demo program Free (20 segments, also runs ARRL models)
  - EZNEC v.6 \$100 (500 segments)
  - EZNEC+ v.6 \$150 (2,000 segments)f
  - EZNEC Pro/2 v.6 \$525 (45,000 segments)
  - EZNEC Pro/4 v.6 \$675 (sold only to NEC4 licensees)
- **4nec2** <https://www.qsl.net/4nec2>
  - GUI runs NEC2 and NEC4, 11,000 segments, two optimizers, all NEC commands supported. Free.
- **MININEC** <http://www.w8io.com/mininec.htm>  
or <https://www.blackcatsystems.com/software/mininec-antenna-analysis-modeling-software.html>
  - Black Cat Systems offers MiniNEC Pro version 1.4.0, \$29
- **MMANA-GAL** <https://hamsoft.ca/pages/mmana-gal.php>
  - Free Basic version 8,192 segments. Pro version 32,000 segments, \$130
- **WIPL-D** <https://wipl-d.com>
  - Old version "Microwave Lite" v6.0 (665 unknowns), or free 30-day trial of professional v17
- **FEKO Student Edition** <http://altairuniversity.com/feko-student-edition>
- **CST Studio Suite Student Edition** <https://www.3ds.com/products-services/simulia/products/cst-studio-suite/student-edition>
  - 3D 10,000 tet voxels, 9 example modeling files and tutorial videos. Free basic version.
- **Meep** <https://meep.readthedocs.io/en/latest>
  - Open-source FDTD modeling code. Uses Python or Scheme. Free.
- **openEMS** <http://openems.de/start>
  - Open-source FDTD modeling code. Uses Matlab or Octave. Free.
- **MEFiSTo** <http://www.faustcorp.com>
  - FDTD modeling code. Free 2D basic version.

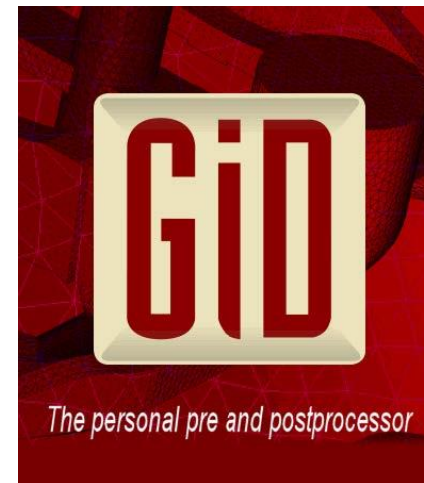
After Dec. 31, 2021,  
EZNEC will be free but unsupported  
Pro/4 will no longer be available

# HOBBIES



Y. Zhang, et al., *Higher Order Basis Based Integral Equation Solver [HOBBIES]*, Wiley, 2012

- Make sure to buy a new copy with software license registration code intact and unused
- Contact HOBBIES Support for license assistance at <https://www.hobbies-em.com>
- Download the GiD v9 and v10 user manuals <https://www.gidhome.com>

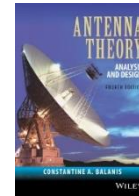
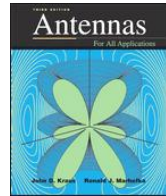




# Accessory Software

-  OPTENNI <https://www.optenni.com>
  - Free trial on request
- **Ampsa Impedance Matching Wizard**  
<https://www.ampsa.com/c/imw-technical-overview>
- **AutoEZ by Dan Maguire, AC6LA, <https://www.ac6la.com>**
  - Recommended accessory software for EZNEC users
  - Excel/Visual Basic program
    - Demo version, free (30 segment limit)
    - Regular version, \$79
  - Requires Excel and EZNEC installed on computer
  - Controls EZNEC to make multiple runs
    - It's a GUI for a GUI for NEC
  - Optimizer – Nelder-Mead algorithm
  - Reads NEC, AO, and MMANA-GAL files
  - Doesn't work with EZNEC-ARRL or EZNEC Demo
  - Replaces MultiNEC, which is no longer available

# Favorite Antenna Books



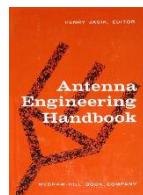
- **Books for antenna engineers and students**

- C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016
- R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011
- J.D. Kraus and R.J. Marhefka, *Antennas*, 3e, McGraw-Hill, 2001

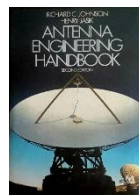
- **Antenna research papers**

- IEEE Xplore subscription online archive, <https://ieeexplore.ieee.org/Xplore/home.jsp>
- Allerton Antenna Applications Symposium DVD archive 1952-2018
- ACES Journal Archives <http://www.aces-society.org/journal.php>
- Progress in Electromagnetics Research <https://www.jpier.org>

- **Antenna Engineering Handbooks – 5 editions**



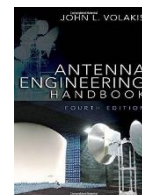
1961



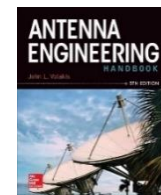
1984



1993

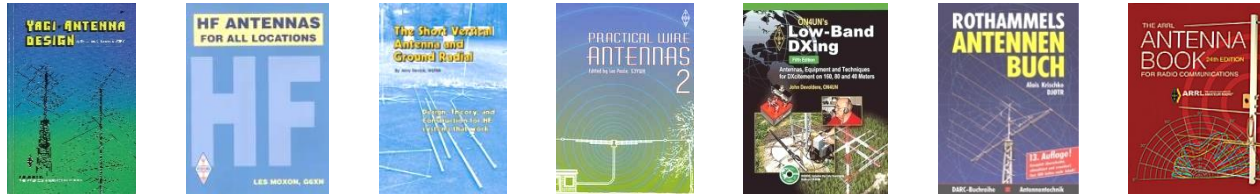


2007



2019

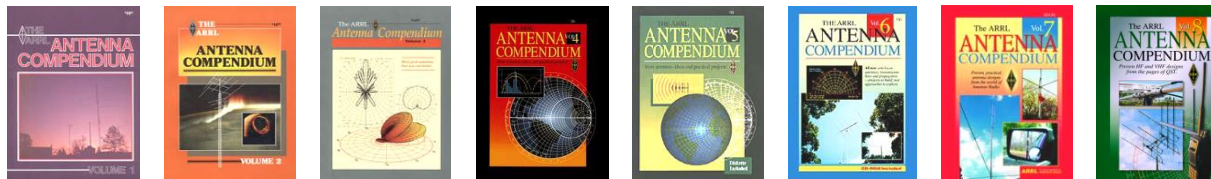
# Favorite Antenna Books continued



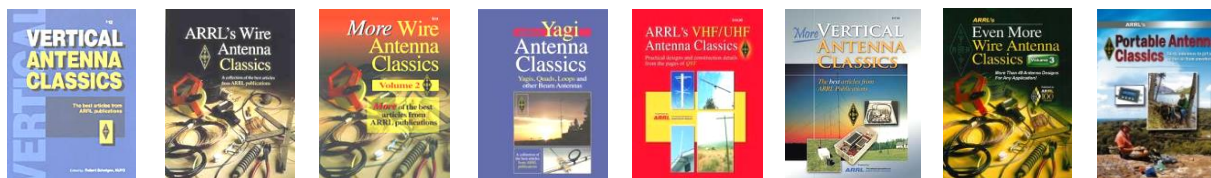
## ■ Books for Radio Amateurs

- H.W. Silver, N0AX, ed., *ARRL Antenna Book*, 24e, ARRL, 2019
- A. Krischke, DJ0TR, ed., *Rothammel's Antenna Book*, English, DARC, 2019
- J. Devoldere, ON4UN, *ON4UN's Low-Band DXing*, 5e, ARRL, 2011
- I. Poole, G3YWX, ed., *Practical Wire Antennas 2*, RSGB, 2005
- J. Sevick, W2FMI, *The Short Vertical Antenna and Ground Radial*, CQ, 2003
- L. Moxon, G6XN, *HF Antennas for All Locations*, 2e, RSGB, 1983
- J.L. Lawson, W2PV, *Yagi Antenna Design*, ARRL, 1986

## ■ ARRL Antenna Compendium series – eight volumes



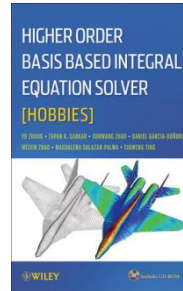
## ■ ARRL Antenna Classics series – eight titles



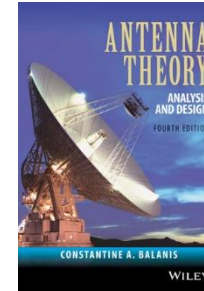
# Recent Antenna Books of Interest



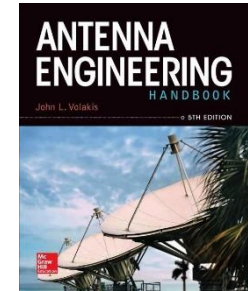
**B.M. Kolundžija and A.R. Djordjević, *Electromagnetic Modeling*, Artech, 2002**



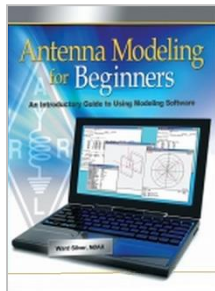
**Y. Zhang et al., *Higher Order Basis Based Integral Equation Solver (HOBBIES)*, Wiley, 2012**



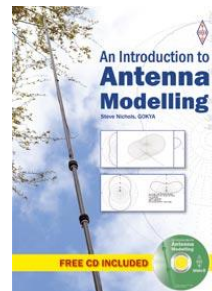
**C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016**



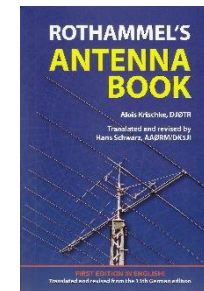
**J.L. Volakis, ed., *Antenna Engineering Handbook*, 5e, McGraw-Hill, 2019**



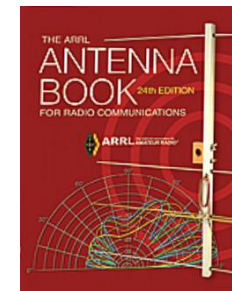
**H.W. Silver, N0AX, *Antenna Modeling for Beginners*, ARRL, 2012**



**S. Nichols G0KYA, *An Introduction to Antenna Modelling*, RSGB, 2014**

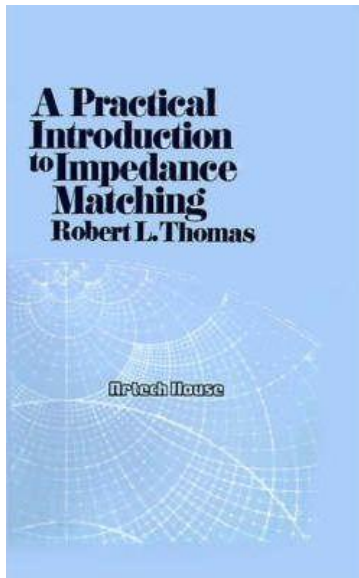


**A. Krischke, DJ0TR, ed., *Rothammel's Antenna Book*, English transl., 13e, DARC, 2019**

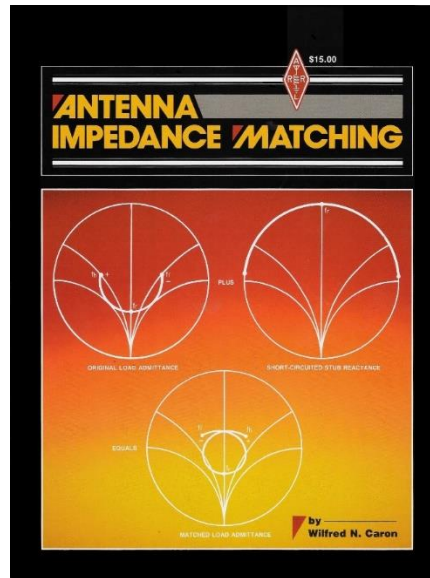


**H.W. Silver, N0AX, ed., *ARRL Antenna Book*, 24e, ARRL, 2019**

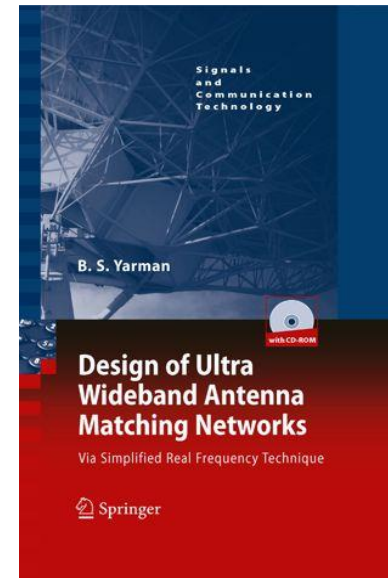
# Impedance Matching from Beginner to Professional



R.L. Thomas, *A Practical Introduction to Impedance Matching*, Artech House, 1976



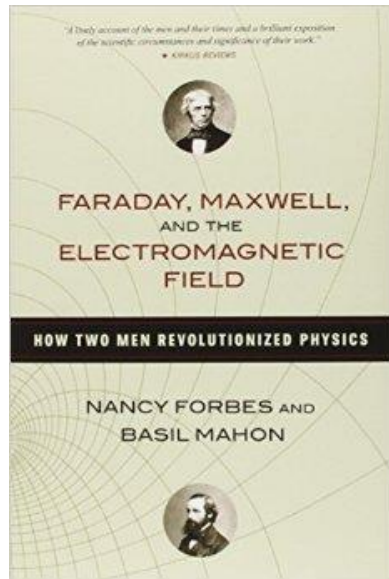
Wilfred N. Caron, *Antenna Impedance Matching*, ARRL, 1989



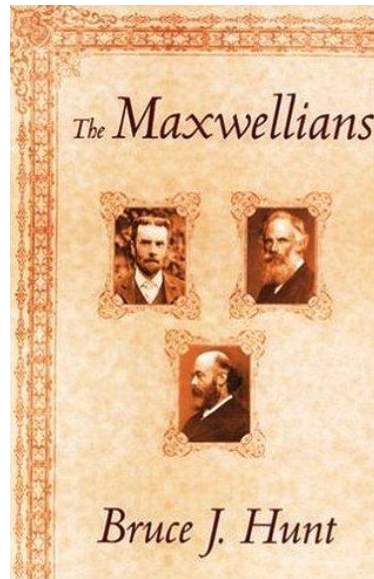
B.S. Yarman, *Design of Ultra Wideband Antenna Matching Networks*, Springer, 2008



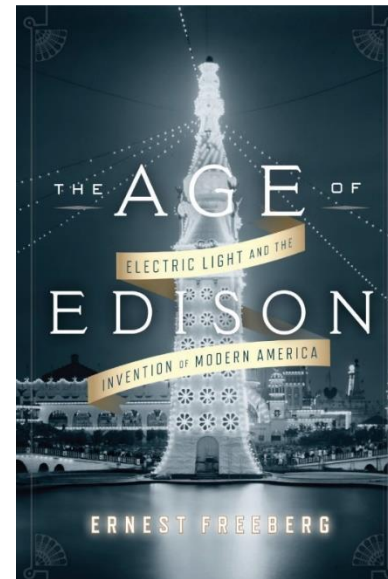
# Four Good History Reads



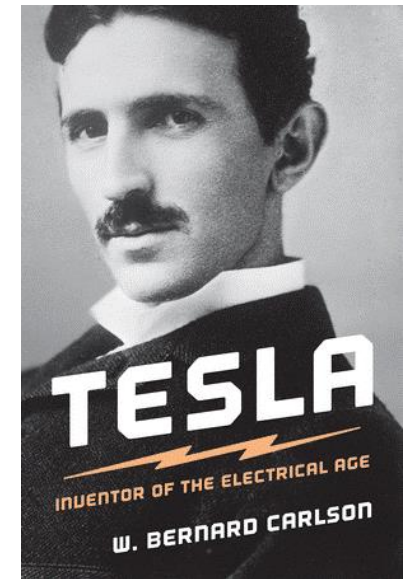
Nancy Forbes and Basil Mahon, *Faraday, Maxwell, and the Electromagnetic Field*, Prometheus, 2014



Bruce J. Hunt, *The Maxwellians*, Cornell University Press, 1991



Ernest Freeberg, *The Age of Edison*, Penguin Books, 2014



W. Bernard Carlson, *Tesla: Inventor of the Electrical Age*, Princeton University Press, 2015

---

# **The End**

**This presentation will be archived at  
<https://www.fars.k6ya.org/docs/k6oik>**